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THEORETICAL AND METHODOLOGICAL FOUNDATIONS OF UNCERTAINTY MODELING IN REAL ESTATE MARKETS

Nonso Izuchukwu Ewurum[1]✉, Njideka Maryclara Aguome[2]✉, Cezary Kowalczyk[3]✉, Idu Robert Egbenta[4]✉, Vivita Pukite[5]✉

[1] ORCID: 0000-0001-6383-5783
[2] ORCID: 0000-0002-6668-9731
[3] ORCID: 0000-0004-3456-7890
[4] ORCID: 0000-0002-4532-2463
[5] ORCID: 0000-0002-9210-0337
[1,2] University of Botswana
Gaborone, **Botswana**
[3] University of Warmia and Mazury in Olsztyn
2 Oczapowskiego Street, 10-719 Olsztyn, **Poland**
[4] University of Nigeria
Nsukka, Enugu State, **Nigeria**
[5] Latvia University of Life Science and Technologies
2 Lielā Street, Jelgava, LV-3001, **Latvia**

ABSTRACT

Motivation: The need to improve the accuracy and reliability of market valuation and risk assessment in real estate markets, especially under conditions of uncertainty.
Aim: To integrate theoretical foundations and methodological approaches for modeling aleatoric and epistemic uncertainties in real estate markets using credal networks and confidence boxes (c-boxes).
Approach: This paper presents a comprehensive theoretical and methodological framework for uncertainty modeling in real estate markets, focusing on the application of credal networks and confidence boxes. It does not include empirical validation or practical case studies, instead providing a detailed conceptual and me – thodological discussion.
Results: The proposed method demonstrates significant improvements in uncertainty quantification and market analysis accuracy in theoretical terms, offering valuable insights for investors, urban planners, and policymakers. However, empirical validation is suggested for future research to confirm practical applicability.

Keywords: aleatoric uncertainty, confidence boxes (c-boxes), credal networks, epistemic uncertainty, real estate market valuation, uncertainty modeling

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl, ✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

INTRODUCTION

The real estate market plays a pivotal role in the global economy, serving as a significant indicator of economic health and a critical component of individual and institutional investment portfolios. Its multifaceted nature encompasses residential, commercial, and industrial properties, each influenced by a complex interplay of economic, social, and political factors. Despite its importance, the real estate market is inherently characterized by various forms of uncertainty, posing substantial challenges to accurate valuation, risk assessment, and informed decision-making (Qu et al., 2023).

Uncertainty in real estate markets arises from numerous sources and can be broadly classified into two categories: aleatoric uncertainty and epistemic uncertainty. Aleatoric uncertainty, also known as statistical or inherent uncertainty, stems from the intrinsic randomness and variability within the market. This includes unpredictable fluctuations in economic indicators, sudden changes in market demand and supply, and unforeseen events such as natural disasters or economic crises. On the other hand, epistemic uncertainty originates from incomplete knowledge or information about the market. Factors contributing to epistemic uncertainty include limited or unreliable data, measurement errors, and subjective judgments in property valuation and forecasting.

Effectively modeling and managing these uncertainties are crucial for stakeholders across the real estate sector, including investors, developers, policymakers, and financial institutions. Accurate uncertainty modeling facilitates better risk management, enhances investment strategies, informs policy formulation, and contributes to the stability and efficiency of real estate markets (Dai & Sheng, 2021).

Traditional probabilistic models, such as Bayesian and Markov models, have been widely employed to address uncertainties in various domains, including real estate. Bayesian models leverage prior knowledge and observed data to update beliefs and make probabilistic inferences, while Markov models focus on modeling stochastic processes and state transitions over time (Hong et al., 2023). While these models offer valuable insights, they often assume precise probability distributions and complete information, which may not be available or realistic in complex and dynamic real estate markets. Such assumptions can lead to oversimplified analyses and potentially misleading conclusions, particularly when dealing with sparse data or highly volatile market conditions.

To overcome these limitations, there is a growing interest in advanced uncertainty modeling techniques that can accommodate imprecise and incomplete information. Among these, credal networks and confidence boxes (c-boxes) have emerged as promising methodologies.

Credal networks are a generalization of Bayesian networks based on the theory of imprecise probabilities. Unlike traditional Bayesian networks that utilize precise probability values, credal networks allow for sets of probability distributions to represent uncertainty (Cortes-Ciriano & Bender, 2018). This flexibility enables the modeling of both aleatoric and epistemic uncertainties more effectively, capturing the range of possible outcomes and acknowledging the presence of incomplete or ambiguous information. In the context of real estate markets, credal networks can model complex dependencies among various market factors, such as economic indicators, demographic trends, and regulatory changes, while accommodating the inherent uncertainties associated with these factors.

Confidence boxes (c-boxes) extend traditional confidence intervals by providing bounds on cumulative distribution functions over a range of confidence levels. C-boxes offer a robust framework for representing estimation uncertainty, particularly useful when dealing with small sample sizes or uncertain data. In real estate applications, c-boxes can be employed to quantify uncertainties in property valuations, rental yields, and investment returns, offering a comprehensive view of potential risks and facilitating more informed decision-making.

The integration of credal networks and c-boxes presents a comprehensive methodological framework for uncertainty modeling in real estate markets

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl,
✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

(Kapon et al., 2024). This integrated approach leverages the strengths of both methodologies, enabling the representation and analysis of complex uncertainties with greater accuracy and reliability. By accommodating both types of uncertainties and incorporating imprecise probabilities, the combined use of credal networks and c-boxes enhances the robustness of market analyses and supports more resilient investment and policy strategies (Marques, 2023).

The purpose of this paper is to explore and establish the theoretical and methodological foundations of uncertainty modeling in real estate markets through the integration of credal networks and confidence boxes. The study aims to demonstrate how these advanced methodologies can address the shortcomings of traditional probabilistic models, providing a more nuanced and effective approach to capturing and analyzing uncertainties inherent in real estate markets.

The paper is structured as follows: Section 2 provides a comprehensive literature review, examining existing approaches to uncertainty modeling in real estate and highlighting the emerging applications of credal networks and c-boxes. Section 3 delves into the theoretical foundations of imprecise probabilities, credal networks, and confidence boxes, elucidating their principles and relevance to real estate uncertainty modeling. Section 4 outlines the proposed methodological framework, detailing the processes of constructing and integrating credal networks and c-boxes for comprehensive uncertainty analysis. Section 5 discusses potential applications and implications of the proposed framework for various stakeholders in the real estate sector. Section 6 addresses the limitations of the study and suggests avenues for future research. Finally, Section 7 concludes the paper by summarizing the key findings and contributions to the field of real estate economics and uncertainty modeling.

Through this exploration, the paper seeks to contribute to the advancement of uncertainty modeling practices in real estate markets, offering a robust and adaptable framework that can enhance analytical precision and support more effective decision-making in the face of complex and multifaceted uncertainties.

LITERATURE REVIEW

The study of uncertainty in real estate markets has garnered increasing attention due to the complex and dynamic nature of these markets. Real estate is influenced by a multitude of factors, including economic conditions, demographic shifts, regulatory changes, and environmental factors (Mustafa et al., 2024). Each of these variables introduces a degree of uncertainty, making accurate market predictions and valuations particularly challenging. Understanding and modeling these uncertainties are essential for improving decision-making processes for investors, policymakers, and other stakeholders.

Uncertainty in real estate can be broadly categorized into aleatoric and epistemic uncertainties. Aleatoric uncertainty arises from inherent randomness and variability within the market, such as fluctuations in interest rates or sudden changes in demand and supply (Quoc et al., 2023). Epistemic uncertainty, on the other hand, is related to the lack of knowledge or incomplete information about market conditions, often exacerbated by limited data availability or unreliable information.

Traditional Approaches to Uncertainty Modeling

Historically, uncertainty in real estate markets has been addressed using traditional probabilistic models, with Bayesian networks and Markov models being among the most prominent.

Bayesian Networks

Bayesian networks (BNs) have proven to be highly effective in various domains, including real estate, where their probabilistic nature supports complex decision-making under uncertainty. BNs allow for the integration of diverse variables, making them invaluable in areas such as property valuation, risk assessment, and market prediction. A comprehensive bibliometric review on Bayesian networks suggests that these models significantly improve decision-making practices across industries, particularly

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl,
✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

in risk management (Juliani & Maciel, 2024). BNs have also been widely adopted in healthcare for disease diagnosis and prognosis, demonstrating their adaptability and predictive accuracy, which exceeds 75% in many cases (Polotskaya et al., 2024). This predictive power is similarly harnessed in real estate markets to anticipate market trends and property values by incorporating economic and environmental data. Furthermore, the application of BNs in risk analysis provides a robust framework to model the uncertainties associated with real estate investments, helping stakeholders make informed decisions despite evolving market conditions (Hanea et al., 2022). While BNs were initially applied in fields like ecology and biology to predict species interactions, their potential in guiding real estate predictive models is becoming more apparent, improving the reliability and interpretability of market analyses (Alday et al., 2022). These insights highlight the versatility of Bayesian networks in optimizing real estate decision-making processes and underscore the need for further research to refine these models for more accurate market predictions.

Markov Models

Markov models have been widely used to model state transitions in various domains, including real estate markets, where they assist in predicting how properties may shift between valuation categories over time. Their strength lies in capturing temporal dynamics and providing a clear framework for analyzing state changes in complex systems like real estate markets. However, Markov models are not without their limitations. As DelSole (2000) points out, these models struggle with short-term prediction accuracy due to their reliance on predefined probability estimates. This challenge becomes particularly pronounced in real estate markets, where external factors such as economic fluctuations or sudden changes in consumer behavior introduce levels of uncertainty that Markov models fail to capture fully. Similarly, while Markov switching models have shown some success in accounting for the non-stationary nature of real estate value indices, their performance still hinges on accurate probability inputs, which can be difficult to estimate in volatile markets (Golomoziy et al., 2024). Recent advancements in combining Markov models with other machine learning techniques, such as temporal encoding, have shown promising results in improving predictive accuracy, achieving over 90% accuracy in real estate value prediction (Jiang et al., 2021). However, despite these improvements, the inherent challenge remains: Markov models are best suited for stable systems with reliable data, and their application in highly uncertain environments like real estate markets requires careful consideration of their limitations.

Emerging Approaches: Credal Networks and Confidence Boxes

In response to the limitations of traditional probabilistic models, researchers have explored alternative frameworks like credal networks and confidence boxes (c-boxes), both rooted in the theory of imprecise probabilities. Credal networks extend Bayesian networks by allowing for sets of probability distributions rather than a single one, providing a more flexible approach for uncertainty modeling in situations where precise probabilities are not available. Similarly, confidence boxes (c-boxes) offer a visual representation of imprecise probabilities by bounding possible outcomes, allowing for a more comprehensive treatment of uncertainty. These methods address key limitations of traditional probabilistic models, particularly in complex systems where data is sparse or uncertain, as seen in partially observable Markov decision processes (POMDPs) where imprecise probabilities play a crucial role in robust decision-making (Bovy et al., 2024). Credal networks have been instrumental in domains requiring robust decision frameworks under uncertainty, while confidence boxes have been used in applications like video anomaly detection, where they simplify the problem space without sacrificing performance (Siemon et al., 2024). By incorporating these more adaptive methods, researchers can better model real-world uncertainties, expanding the applicability

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl, ✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

of probabilistic models in various domains, from AI to corporate network security. Two such methods are credal networks and confidence boxes (c-boxes), both of which are grounded in the theory of imprecise probabilities.

Credal Networks

Credal networks extend Bayesian networks by allowing for sets of probability distributions, known as credal sets, instead of single-point estimates. This framework, grounded in the theory of imprecise probabilities, provides a more nuanced representation of uncertainty, particularly useful in fields like real estate market analysis. By incorporating both aleatoric (data-dependent) and epistemic (knowledge-related) uncertainties, credal networks are particularly well-suited for modeling the interplay of complex variables such as economic indicators, property prices, and regulatory changes. This approach allows for a more flexible and robust analysis of market trends, helping stakeholders navigate uncertain conditions and make more informed decisions. In particular, the real estate market's dynamics, influenced by factors like urban renewal and macroeconomic conditions, make credal networks valuable for risk assessment and opportunity evaluation (Gofman et al., 2024). The method also accommodates the unpredictability of external shocks, such as war or economic recessions, which significantly impact real estate prices and market stability (Yavorska & Shynkarenko, 2024). The ability of credal networks to represent uncertainty through multiple probability distributions offers a strategic advantage, making them a robust tool for analyzing real estate markets in volatile environments.

Confidence Boxes (C-Boxes)

Confidence boxes, or c-boxes, offer a robust and comprehensive method for uncertainty modeling by encoding uncertainty at all confidence levels, making them particularly useful in real estate markets where data is often incomplete or subject to variability. This method extends the traditional concept of confidence intervals, allowing for a more dynamic representation of potential outcomes across a range of confidence levels. In real estate, where accurate risk assessment and property valuation are critical yet often challenged by market fluctuations, c-boxes provide a systematic framework to incorporate estimation uncertainty. For instance, c-boxes have been applied effectively in flood risk assessment models to account for varying levels of risk, improving decision-making in investment and insurance sectors (Pavesi et al., 2024). Similarly, c-boxes have proven valuable in broader real estate risk assessment, such as modeling economic losses due to climate change, where uncertainty is particularly high (Peng et al., 2024). By offering a detailed and reliable picture of uncertainty, c-boxes enhance the accuracy of market analyses, investment forecasts, and risk evaluations in the real estate sector, providing stakeholders with tools to manage and mitigate risks more effectively.

Empirical Applications and Case Studies

The empirical application of credal networks and c-boxes in real estate markets is still in its early stages, but growing interest in these methods has led to a number of promising studies. Ding et al. (2022) applied credal networks to model uncertainties in hazardous material transportation, demonstrating the method's applicability to environments characterized by significant uncertainty. Although not directly related to real estate, this study highlights the potential of credal networks in complex, uncertain contexts.

Morais et al. (2022) employed c-boxes in human reliability analysis, showcasing their utility in scenarios where data is sparse and uncertainty is high. The insights from these applications can be adapted to the real estate market, where similar challenges exist.

While empirical applications of credal networks and c-boxes in real estate are limited, the existing literature suggests that these methods hold significant promise for improving the accuracy and reliability of market analyses. As research in this area expands, it is expected that more case studies and applications will emerge, further validating the utility of these advanced modeling techniques.

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl,
✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

Gaps in the Literature and Future Directions

Despite advancements in uncertainty modeling, the integration of credal networks and confidence boxes (c-boxes) in real estate markets remains underexplored, particularly in emerging markets. These models offer significant potential for improving uncertainty representation, yet empirical studies validating their application in real-world real estate contexts are limited. Recent studies, such as Zhang et al. (2024), demonstrate the effectiveness of machine learning methods like dynamic neural networks for real estate mass appraisal, which can serve as a foundation for integrating more sophisticated uncertainty models like credal networks and c-boxes. Additionally, research by Galante et al. (2024) highlights the challenges of data scarcity in the real estate sector, further complicating the application of complex models. While machine learning and artificial neural networks (ANNs) have shown promise in improving appraisal accuracy, they face challenges related to computational complexity and data requirements. Therefore, future research should focus on developing efficient computational algorithms that can handle the intricacies of credal networks and c-boxes while ensuring they are applicable across diverse real estate markets, particularly those with incomplete or highly variable data.

Furthermore, the potential integration of machine learning techniques with credal networks and c-boxes represents a promising avenue for enhancing predictive capabilities in real estate analysis. Such integration could lead to more accurate and dynamic models, capable of adapting to rapidly changing market conditions.

The literature on uncertainty modeling in real estate markets highlights a growing shift from traditional probabilistic methods to more flexible frameworks such as credal networks and confidence boxes (c-boxes). These methods offer advanced tools for capturing uncertainties in dynamic and complex environments, where precise probability estimates are often unattainable. Recent studies, such as those by Zhang et al. (2024), have demonstrated the importance of integrating machine learning and advanced uncertainty models to enhance real estate market predictions and mass appraisals. While empirical research validating these models remains sparse, emerging methodologies like gradient boosting and neural networks show promise in real estate decision-making, particularly in mitigating risks related to market volatility and incomplete data (Peng et al., 2024). As these frameworks gain traction, future research should focus on their practical application, especially in diverse market conditions where uncertainty plays a critical role. By leveraging the nuanced uncertainty modeling capabilities of credal networks and c-boxes, stakeholders can make more informed decisions, thereby improving the robustness of market analyses.

Integrated Use of Credal Networks and C-Boxes in Real Estate Markets: Advantages and Applications

The integrated use of credal networks and confidence boxes (c-boxes) in real estate markets presents several advantages, particularly in improving risk assessment, enhancing decision-making, and adapting to emerging markets. Credal networks, by capturing both aleatoric (data-related) and epistemic (knowledge-based) uncertainties, enable more comprehensive risk assessments, offering a detailed understanding of the potential risks associated with various market conditions and investment decisions (Peng et al., 2024). This model also supports enhanced decision-making by providing investors, developers, and policymakers with reliable insights, especially in data-limited environments. For instance, in contexts where market data is unreliable or incomplete, c-boxes can be used to model the full spectrum of uncertainty, making the approach particularly useful in emerging markets (Qin & Li, 2024). Traditional models often fail in these regions due to their reliance on robust datasets, but the flexibility of credal networks and c-boxes allows for more resilient market predictions. As these models are further developed and tested

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl, ✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

in empirical research, their adaptability will continue to offer significant improvements in real estate market analysis and investment strategies.

Conclusion on Uncertainty Modeling in Real Estate Markets

The theoretical foundations of uncertainty modeling in real estate markets, particularly through the integration of credal networks and confidence boxes (c-boxes), provide a more robust and adaptable framework for navigating the complexities of market dynamics. These methodologies, based on the theory of imprecise probabilities, offer significant advancements over traditional probabilistic models by capturing both aleatoric and epistemic uncertainties. This approach allows for a more nuanced analysis of market trends, risk factors, and investment decisions. Studies such as Basili & Pratelli (2024) highlight the effectiveness of interval probability measures in addressing uncertainty, while Chen et al. (2024) demonstrate how economic environment uncertainties influence financialization strategies in real estate firms. Additionally, the dynamic analysis of residential real estate markets underscores the value of integrating machine learning and deep learning to better understand the multifaceted relationships between economic indicators and market performance (Chan et al., 2024). These theoretical foundations not only push the field of real estate economics forward but also provide stakeholders with the necessary tools to navigate market uncertainties with greater precision and confidence.

METHODOLOGY

The primary objective of this section is to outline the theoretical and methodological framework that underpins the modeling of uncertainties in real estate markets. The methodology employed in this study is rooted in the integration of credal networks and confidence boxes (c-boxes), both of which are derived from the theory of imprecise probabilities. This approach offers a comprehensive way to address the complex uncertainties inherent in real estate markets, where both aleatoric (inherent randomness) and epistemic (knowledge-based) uncertainties are prevalent. By providing a detailed theoretical procedure for constructing and applying these models, this methodology equips stakeholders with the tools needed to navigate the complexities of real estate markets more effectively. While this paper focuses on the theoretical foundations and methodological framework, future research could involve empirical validation and case studies to further demonstrate the practical applicability of this approach.

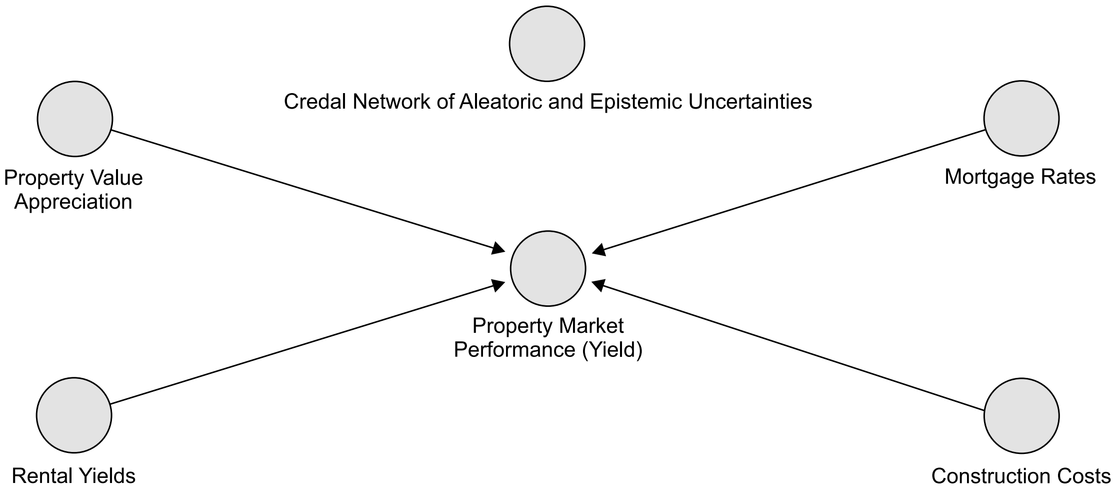

Fig. 1. Credal Network of Aleatoric and Epistemic Uncertainties
Source: Authors' Design.

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl,
✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

To illustrate the application of credal networks to the uncertainties on property market, we constructed a simplified model as follows (Fig. 1):

Credal Networks: Construction and Application

Credal networks are an extension of Bayesian networks that allow for the representation of uncertainty through sets of probability distributions rather than single-point estimates. This flexibility makes them particularly suitable for modeling the complex dependencies between variables in real estate markets, where uncertainty is often high and data is incomplete.

Defining Variables and Relationships

The first step in constructing a credal network is identifying the key variables that influence real estate markets. These typically include:
– Property prices,
– Rental yields,
– Construction costs,
– Interest rates,
– Economic indicators (e.g., GDP growth, inflation rates),
– Regulatory factors.

Once these variables are defined, the relationships between them must be established. These relationships are represented as edges in the credal network, indicating conditional dependencies. For example, property prices might depend on economic indicators, while rental yields could be influenced by both property prices and construction costs.

Representing Uncertainty with Credal Sets

For each relationship between variables, conditional probability distributions (CPDs) are defined using credal sets. Unlike traditional Bayesian networks that require precise CPDs, credal networks allow for the specification of intervals or sets of probabilities, reflecting the uncertainty inherent in the data.

Mathematically, a credal set K for a variable X given its parent variables $Pa(X)$ can be represented as: $K(X \mid Pa\ (X)) = \{P(X \mid Pa\ (X)) \colon P$ is a valid probability distribution$\}$.

This set-based approach enables the model to capture a range of possible outcomes rather than committing to a single, potentially inaccurate, probability estimate. This is particularly valuable in real estate markets, where data may be scarce or unreliable, and where the future is inherently uncertain.

Propagation of Uncertainty in the Network

Once the credal network is constructed, it can be used to propagate uncertainty through the network, allowing for the analysis of how changes in one variable (e.g., an increase in interest rates) affect others (e.g., property prices, rental yields). This is done by calculating the lower and upper bounds of the probability distributions for each variable, considering the credal sets associated with their parent variables.

RESULTS

Confidence boxes (c-boxes) provide a robust method for representing estimation uncertainty in the parameters of the credal network. They are particularly useful in scenarios where data is sparse, such as in emerging real estate markets, or where the available data is subject to significant uncertainty.

Parameter Estimation with Confidence Intervals

The first step in constructing a c-box is estimating the parameters for each variable in the credal network using available data. For example, the mean and variance of property value appreciation rates can be estimated from historical data. However, instead of providing a single estimate with a fixed confidence interval, a c-box encapsulates the range of possible values at various confidence levels.

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl, ✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

A c-box $C(\theta)$ for a parameter θ is constructed by determining the lower and upper bounds of the parameter at different confidence levels: $C(\theta) = \{[L(\alpha), U(\alpha)]: \alpha \in [0,1]\}$ where $L(\alpha)$ and $U(\alpha)$ are the lower and upper bounds of the cumulative distribution function (CDF) for θ at confidence level α.

Incorporating C-Boxes into Credal Networks

C-boxes can be integrated into credal networks to represent the uncertainty in the parameters that define the network's conditional probability distributions. This dual approach ensures that both aleatoric and epistemic uncertainties are fully accounted for in the model.

For example, if the appreciation rate of property values is a node in the credal network, the uncertainty in this rate can be captured by a c-box, which provides a range of possible values rather than a single point estimate. This c-box is then used to inform the credal set for the property value node, ensuring that the network reflects the full scope of uncertainty.

To derive marginal probabilities from the credal network with c-boxes, we sum over the joint credal sets (1):

$$P(X_i) = \sum_{X_{-i}} K\big(X_i \big| Pa, (X_i)\big) \quad (1)$$

Where X_{-i} denotes all variables except X_i.

For a given query variable Q and evidence E, the bounds on the marginal probability can be determined by (2) (3):

$$minP(Q|E) = min_K \sum_{Q \cup E} \prod_{i=1}^{n} P(X_i|Pa(X_i)) \quad (2)$$

$$maxP(Q|E) = max_K \sum_{Q \cup E} \prod_{i=1}^{n} P(X_i|Pa(X_i)) \quad (3)$$

Figure 2 illustrates the construction of a c-box for a given parameter.

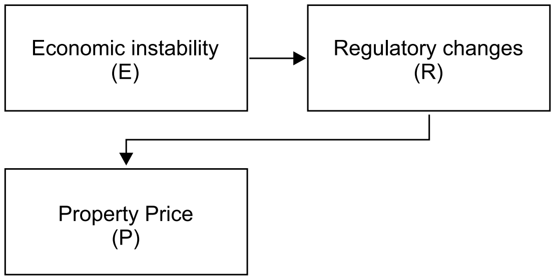

Fig. 2. A Simple Model of Market Volatility
Source: Authors' design.

Simulation and Inference

Once the credal network and c-boxes are constructed, the integrated model can be used to simulate various market scenarios and infer the probability distributions of key market outcomes under different conditions.

Scenario Analysis

The model allows for the exploration of various "what-if" scenarios, such as:
– How would a sudden increase in interest rates impact property prices and rental yields?
– What are the potential effects of regulatory changes on market stability?
– How does economic growth or contraction influence construction costs and investment returns?

By simulating these scenarios, the model provides a range of possible outcomes, along with the associated uncertainties, allowing stakeholders to better understand and prepare for different market conditions.

Inference and Decision-Making

The lower and upper bounds of the probability distributions generated by the model can be used to inform decision-making. For instance, investors can assess the risk and potential return of different investment strategies, policymakers can evaluate the likely impact of regulatory changes, and developers can plan projects with a clearer understanding of the potential risks and opportunities.

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl,
✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

Propose empirical validation

This section outlines a proposed framework for the empirical validation of the methodology described in this study. The aim is to demonstrate the potential effectiveness of integrating credal networks and confidence boxes (c-boxes) in modeling and quantifying uncertainties in real estate markets through a future case study.

To validate the proposed methodology, it is suggested to utilize a combination of empirical data and expert opinions to construct a comprehensive dataset for analysis. The proposed data collection process includes:

1. Empirical Data: collect historical market data on property values, rental yields, construction costs, and mortgage rates from various sources such as government registries, real estate firms, and financial institutions. This data would provide the foundation for modeling aleatoric uncertainties.
2. Expert Opinions: conduct expert surveys to gather insights on market conditions, regulatory impacts, and economic factors influencing the real estate market. These surveys would help capture epistemic uncertainties, particularly in areas where empirical data is scarce or incomplete.
3. Credal Networks – the construction of credal networks as part of the empirical validation would involve the following steps:
   – Defining Variables: identify key market variables such as property values, rental yields, construction costs, and mortgage rates. Each variable would be treated as a node in the credal network.
   – Establishing Dependencies: map the relationships between variables based on empirical data and expert knowledge. These dependencies would be represented by edges in the network.
   – Conditional Probability Distributions (CPDs): define CPDs for each variable using credal sets, allowing for intervals or sets of probabilities instead of single-point estimates. This approach captures both aleatoric and epistemic uncertainties.
4. Confidence Boxes (C-Boxes):

The construction of c-boxes for representing parameter uncertainty would involve:

– Parameter Estimation: estimate parameters for each market variable using available empirical data. For instance, calculate the mean and variance of property value appreciation rates.
– Confidence Intervals: construct confidence intervals at multiple confidence levels for each parameter. Determine the lower and upper bounds of the parameter estimates.
– C-Box Construction: combine the confidence intervals at different levels to form a c-box, encapsulating the estimation uncertainty of the parameter. This approach provides a more comprehensive representation of uncertainty compared to traditional single-level confidence intervals.

5. Integration of Credal Networks and C-Boxes:

The proposed integration of credal networks and c-boxes would involve:

– Modeling Dependencies: use credal networks to model the dependencies between market variables, incorporating both aleatoric and epistemic uncertainties.
– Parameter Uncertainty Representation: employ c-boxes to represent the uncertainty in the parameters of the credal networks. This dual approach ensures that both types of uncertainties are adequately captured and quantified.
– Simulation and Inference: perform simulations and infer the probability distributions of market outcomes under different scenarios. Calculate the lower and upper probabilities for various outcomes, providing a range of plausible scenarios.

The proposed empirical validation framework outlines the steps necessary to test the effectiveness of integrating credal networks and c-boxes in real estate market analysis. By following this framework, future research can provide robust empirical evidence to support the theoretical and methodological foundations presented in this study, ultimately enhancing the accuracy and reliability of market valuations and risk assessments.

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl, ✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

DISCUSSION

The integration of credal networks and confidence boxes in uncertainty modeling for real estate markets represents a significant advancement in risk assessment and decision-making. These methodologies allow for a more flexible and comprehensive approach to understanding and managing market uncertainties. Credal networks offer the ability to handle imprecise probabilities, while confidence boxes capture uncertainty across multiple levels, providing a more nuanced analysis of market dynamics. The practical implications of these tools are substantial, offering investors, policymakers, and real estate professionals the capacity to make more informed decisions in environments where data may be limited or unreliable (Peng et al., 2024). Despite the challenges associated with implementing these advanced models, such as computational complexity and data requirements, their adaptability and ability to provide comprehensive risk assessments make them invaluable in emerging and volatile markets (Qin & Li, 2024). The flexibility of credal networks and confidence boxes offers a pathway to improved decision-making processes, enabling stakeholders to navigate uncertainties with greater confidence and precision.

Future research and empirical validation will be critical in further refining this approach and expanding its applicability, ensuring that it continues to provide robust and reliable insights in the complex and dynamic field of real estate economics.

Implications for Real Estate Market Analysis

The integration of credal networks and confidence boxes (c-boxes) into uncertainty modeling offers a significant advancement for analyzing real estate markets. Traditional probabilistic models, while useful, often fail to capture the full extent of uncertainties inherent in these markets. By allowing for imprecise probabilities and a more flexible representation of uncertainty, credal networks and c-boxes provide a robust framework for more accurate market analysis. Recent studies emphasize the increasing role of machine learning and advanced algorithms in refining these models. Zhang et al. (2024) demonstrated how dynamic neural networks could be effectively integrated into mass real estate appraisals, achieving superior performance over traditional models. Similarly, Yang (2023) explored data-driven investment strategies in real estate, revealing the transformative potential of predictive analytics for optimizing decision-making under uncertainty. By capturing a broader range of possible outcomes, these advanced methodologies not only enhance risk assessments but also enable more informed and precise investment strategies in complex and evolving real estate markets.

One of the key implications of credal networks in real estate market analysis is their ability to effectively model both aleatoric and epistemic uncertainties. Aleatoric uncertainty, arising from the inherent randomness and variability of market behavior, can be well captured by credal networks, which model the complex dependencies among variables like economic indicators, regulatory changes, and market demand. This enables a more comprehensive understanding of how these factors interact and influence market outcomes. For instance, the study by Saranathan et al. (2024) highlights the use of advanced neural networks in handling dual uncertainties, demonstrating how these models can provide reliable predictions in dynamic environments. Additionally, Zhang et al. (2024) show how dynamic neural networks can be applied in real estate mass appraisals to address both aleatoric and epistemic uncertainties. These approaches offer valuable insights for real estate investors and decision-makers, helping them navigate uncertain market conditions with greater precision. On the other hand, epistemic uncertainty, which stems from incomplete knowledge or data gaps, is effectively managed by c-boxes. C-boxes allow for a detailed representation of estimation uncertainty, particularly in situations where data is scarce or highly variable. This capability is crucial in real estate markets, where data reliability and availability often pose significant challenges.

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl,
✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

By integrating these two approaches, the proposed methodology provides a comprehensive tool for analyzing real estate markets under conditions of uncertainty. This integration not only enhances the accuracy of market predictions but also supports more informed decision-making by offering a detailed understanding of the risks and potential outcomes associated with various scenarios.

Strengths of the Integrated Approach

The integrated use of credal networks and c-boxes presents several key strengths:

- Flexibility in Uncertainty Representation: The ability to model both precise and imprecise probabilities allows for a more realistic representation of market uncertainties. This is particularly important in real estate markets, where the unpredictability of economic conditions, regulatory environments, and other factors can lead to significant variability in market outcomes.
- Comprehensive Risk Assessment: By capturing both types of uncertainties, the model provides a more complete assessment of the risks associated with different market scenarios. This enables investors, policymakers, and other stakeholders to make better-informed decisions that account for the full range of possible outcomes.
- Enhanced Decision Support: The integrated model supports decision-making by offering a clear visualization of the potential impacts of different variables on market outcomes. For example, the model can simulate how changes in interest rates or property taxes might affect property values and rental yields, providing stakeholders with valuable insights into the potential risks and benefits of different policy decisions or investment strategies.
- Adaptability to Different Market Conditions: The methodology is adaptable to a wide range of market conditions, making it applicable not only to mature markets but also to emerging ones where data may be limited or uncertain. This adaptability ensures that the model remains relevant across different contexts, providing a valuable tool for real estate analysis in diverse environments.

Limitations and Challenges

Despite its strengths, the proposed methodology also faces certain limitations and challenges that must be acknowledged:

- Computational Complexity: The use of credal networks and c-boxes involves significant computational complexity, particularly when dealing with large networks or multiple variables. This complexity can be a barrier to the practical application of the methodology, especially in contexts where computational resources are limited. Future research could focus on developing more efficient algorithms to reduce the computational burden and make the approach more accessible.
- Reliance on Expert Judgment: While the methodology provides a robust framework for modeling uncertainty, it often relies on expert judgment to define credal sets and confidence intervals, particularly in the absence of comprehensive data. This reliance introduces the potential for bias and subjectivity, which could affect the accuracy of the model's predictions. To mitigate this risk, it is important to use rigorous elicitation techniques and, where possible, to combine expert judgment with empirical data.
- Need for Empirical Validation: Although the theoretical foundations of the methodology are strong, there is a need for empirical validation to confirm its practical applicability. Future research should focus on applying the model to real-world case studies and validating its predictions against actual market outcomes. This validation would help to refine the methodology and ensure that it provides accurate and reliable insights for real estate analysis.

Future Research Directions

The discussion of the limitations and strengths of the proposed methodology opens several avenues for future research:

- Empirical Application and Validation: Future studies should apply the integrated approach to real estate markets in different regions and under varying conditions. Empirical validation will be

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl, ✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

crucial in assessing the practical utility of the model and identifying areas where further refinement is needed.
- Development of Computational Tools: There is a need for the development of more efficient computational tools and algorithms that can handle the complexity of credal networks and c-boxes. Advances in this area could make the methodology more accessible to a broader range of users, including those with limited computational resources.
- Integration with Machine Learning: The integration of machine learning techniques with the proposed methodology could enhance its predictive capabilities. Machine learning models could be used to refine the probability distributions within credal networks or to automate the generation of c-boxes, leading to more accurate and dynamic models of real estate markets.
- Exploration of Other Market Factors: Future research could also explore the application of the integrated approach to other factors affecting real estate markets, such as environmental risks, demographic shifts, and technological innovations. This would help to further validate the methodology and expand its applicability to a wider range of real estate challenges.

Practical Implications

The practical implications of this research are significant for various stakeholders in the real estate market:
- Investors: The ability to more accurately quantify and manage uncertainties can lead to better investment decisions and improved portfolio performance. By understanding the full range of potential outcomes, investors can make more informed choices that align with their risk tolerance and investment goals.
- Policymakers: The insights gained from the model can help policymakers develop more effective regulatory frameworks that enhance market stability and transparency. By simulating the potential impacts of different policy decisions, the model provides a valuable tool for evaluating the potential risks and benefits of various regulatory options.
- Real Estate Professionals: Appraisers, analysts, and developers can leverage the methodology to provide more reliable valuations and assessments. This can ultimately lead to more accurate pricing, better risk management, and more informed development strategies, benefiting the broader market.

CONCLUSIONS

This paper has introduced a comprehensive theoretical and methodological framework for modeling uncertainties in real estate markets by integrating credal networks and confidence boxes (c-boxes). The approach presented here addresses the inherent complexities and uncertainties of real estate markets, offering significant improvements over traditional probabilistic models, which often depend on precise probabilities and complete information.

The study builds on the theory of imprecise probabilities, utilizing credal networks and c-boxes to model both aleatoric (random) and epistemic (knowledge-based) uncertainties. This dual approach offers a more flexible and realistic representation of uncertainty, especially critical in real estate markets where data is incomplete, and market conditions are volatile. Credal networks enable the modeling of complex dependencies among variables such as economic indicators, regulatory changes, and market demand, thereby providing a comprehensive analysis of market dynamics and estimation uncertainties. Studies like Pulkkinen et al. (2023) show how Bayesian neural networks with variational inference have been applied to capture both types of uncertainties, underscoring the importance of integrating these models for improved predictive accuracy. Similarly, Zhang et al. (2023) highlight the role of non-parameterized probability boxes (P-boxes) in handling uncertainty propagation, showing that these methods can significantly enhance decision-making in complex systems like real estate. Credal networks are employed to model the probabilistic relationships between key market variables, while c-boxes offer a robust framework for

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl,
✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

quantifying parameter uncertainties across multiple confidence levels. This integration enhances the accuracy and reliability of market analyses, supporting more informed decision-making.

The proposed framework provides substantial value to stakeholders in the real estate sector, including investors, policymakers, and real estate professionals. By offering a more detailed understanding of potential risks and outcomes, this methodology enhances risk management, valuation accuracy, and policy development. For investors, the framework allows for more effective modeling and management of uncertainties, improving investment strategies and portfolio management (Pulkkinen et al., 2023). Policymakers benefit from the insights generated by credal networks and c-boxes, which can inform the development of more robust regulatory frameworks that ensure market stability and transparency (Zhang et al., 2023). Additionally, real estate professionals including appraisers, analysts, and developers can use these tools to improve the accuracy and reliability of their market assessments, enabling more precise pricing, better risk management, and more informed development planning (Zhang et al., 2024). Overall, the practical applications of this approach are wide-ranging and significant, offering stakeholders valuable tools to navigate the complexities of real estate markets with greater confidence.

While the proposed methodology represents a significant advancement in uncertainty modeling, it introduces challenges, particularly in terms of computational complexity, which may hinder its practical application in large-scale analyses or in contexts with limited resources. For instance, convex hull analysis methods have been proposed to address computational limitations in uncertainty quantification (Çatak & Kuzlu, 2024), but their implementation in credal networks and c-boxes remains resource-intensive. Furthermore, the reliance on expert judgment to define credal sets and confidence intervals introduces subjectivity, which can bias model outputs. Research by Yang et al. (2024) highlights the importance of mitigating exploitation bias in learning models, which could be adapted to address bias in expert-based modeling frameworks. Future research should focus on combining empirical data with expert judgment to mitigate these biases and enhance the robustness of credal network applications. Moreover, empirical validation is crucial for confirming the practical utility of these models. Studies such as Lafi and Chammem (2023) emphasize the need for empirical studies to test theoretical frameworks against real-world outcomes, which is a necessary step for ensuring the reliability of credal networks and c-boxes in real estate market analysis.

Looking forward, several avenues for future research emerge from this study. Empirical validation through real-world applications will be critical in assessing the effectiveness of the proposed methodology. The development of more efficient computational tools and algorithms will be essential to make the methodology more accessible and practical for a broader range of users. For example, Kim (2024) emphasizes the need for algorithmic improvements in forecasting real estate markets using AI, while Galante et al. (2024) explore how neural networks can enhance real estate appraisals. Moreover, integrating machine learning techniques with credal networks and c-boxes could enhance predictive capabilities, as seen in studies like Ancy and Praveenchandar (2024), which highlight the predictive accuracy of machine learning in real estate price forecasting. Expanding the application of the methodology to other factors influencing real estate markets such as environmental risks, demographic changes, and technological innovations could provide further insights and validation, as noted in Mueller et al. (2024), who discuss machine learning's role in modeling gentrification and related market dynamics.

In conclusion, the integration of credal networks and confidence boxes represents a significant theoretical and methodological advancement in the modeling of uncertainties in real estate markets. By capturing both aleatoric and epistemic uncertainties, the proposed framework provides a more comprehensive and realistic tool for market analysis. While challenges remain, the potential benefits for investors, policymakers, and real estate professionals are

✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl, ✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

substantial. The findings of this study contribute to the field of real estate economics by offering a robust and adaptable framework for uncertainty modeling. As research and empirical validation continue, this approach is likely to play an increasingly important role in enhancing the precision and reliability of real estate market analyses, ultimately leading to better decision-making and improved market outcomes.

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✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl,
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✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl, ✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv

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✉ewurumn@ub.ac.bw, ✉aguomen@ub.ac.bw, ✉cezary.kowalczyk@uwm.edu.pl,
✉idu.egbenta@unn.edu.ng, ✉vivita.pukite@llu.lv