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UNCERTAINTY IN PROPERTY VALUATION: ALEATORIC AND EPISTEMIC CHALLENGES IN THE NIGERIAN REAL ESTATE MARKET

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ABSTRACT

Motives: Real estate markets, particularly in emerging economies such as Nigeria, are subject to significant uncertainties. These can be broadly categorized into aleatoric uncertainties, which arise from inherent market variability, and epistemic uncertainties, which arise from incomplete knowledge and data gaps. Traditional valuation models often struggle to fully capture these uncertainties, making property investment decisions more challenging.
Aim: This study aims to explore the integration of credal networks and confidence boxes (c-boxes) as an innovative approach to modeling uncertainty in real estate markets. The research focuses on the property market in Enugu, Nigeria, and demonstrates how this methodology improves the accuracy and reliability of market valuations compared to conventional probabilistic models.
Approach: Combining ex post facto analysis with expert surveys, the study applied a hybrid model incorporating credal networks and c-boxes to quantify both aleatoric and epistemic uncertainties. Empirical validation was conducted on property valuation data from the Enugu market, using Monte Carlo simulations and probabilistic sensitivity analysis to assess the model's robustness.
Results: The findings revealed significant aleatoric uncertainty in property value appreciation and construction costs, while epistemic uncertainty was more pronounced in rental yields and mortgage rates. The credal network approach outperformed traditional models, providing an average improvement of 24.09% in valuation accuracy and reliability, thereby offering a more comprehensive framework for decision-making in property markets.

Keywords: aleatoric uncertainty, epistemic uncertainty, credal networks, confidence boxes, emerging markets, property market valuation, Nigeria

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INTRODUCTION

The real estate markets, especially in emerging economies like Nigeria, are characterized by significant uncertainties that complicate property valuation and investment decisions. These uncertainties can be broadly categorized into two types: aleatoric, which arise from inherent market variability (Nevin et al., 2024), and epistemic, stemming from incomplete knowledge or data gaps (Laskin & Cherkesova, 2020). Traditional valuation models, such as Bayesian networks, often fail to adequately capture these uncertainties, making them ill-suited for markets with limited data and rapidly changing conditions (Sobieraj & Metelski, 2022).

In Enugu, Nigeria, the property market exemplifies these challenges. The city, experiencing rapid urbanization and economic growth, presents a complex landscape where property values are influenced by diverse factors such as informal transactions, inconsistent data collection, and evolving regulatory frameworks (Ifeanacho & Egbenta, 2023). This mix of aleatoric and epistemic uncertainties underscores the need for innovative approaches to property valuation. For instance, informal property sales, which are prevalent in many parts of Nigeria, often lack the documentation needed for accurate analysis, contributing to substantial epistemic uncertainty (Cheloti & Mooya, 2022). Similarly, rapid urban development can lead to unpredictable fluctuations in property demand and prices, illustrating the aleatoric aspects of the market (Abaidoo & Agyapong, 2021).

Traditional probabilistic models, such as Bayesian networks, often rely on precise probabilities and comprehensive datasets (Marfatia, 2020; Sobieraj & Metelski, 2022). However, these assumptions are rarely met in emerging markets, leading to oversimplified models that fail to address the distinct nature of uncertainty in these contexts (Cheloti & Mooya, 2022). Such limitations highlight the need for more robust frameworks that can differentiate between aleatoric variability and epistemic gaps (Morais et al., 2021). Moreover, reliance on expert judgment to fill data gaps in traditional models introduces biases, further undermining their reliability (Mauksch et al., 2020).

This study proposes the integration of credal networks and confidence boxes (c-boxes) as a novel methodology for addressing these challenges. Credal networks, based on the theory of imprecise probabilities (Walley, 1991), allow for the representation of a range of plausible probability distributions rather than fixed values. When combined with c-boxes, which encode confidence intervals across all confidence levels (Ferson et al., 2015), this approach provides a comprehensive framework for quantifying both types of uncertainties. The use of these tools not only enhances flexibility in modeling but also aligns with the realities of data limitations and variability commonly encountered in emerging markets (Morais et al., 2021).

The aim of this research is to demonstrate how this hybrid methodology can enhance the precision and reliability of property market valuations in Enugu. By applying credal networks and c-boxes to key market variables (Ewurum et al., 2024) – such as property value appreciation, rental yields, construction costs, and mortgage rates—this study seeks to provide a more accurate and reliable tool for market analysis compared to traditional models. Furthermore, the methodology's reliance on interval-based probabilities ensures that the models remain robust even when input data is incomplete or uncertain.

The implications of these findings are significant for stakeholders in the real estate sector. For investors, this approach offers a clearer understanding of market risks, enabling better portfolio management and decision-making (Marfatia, 2020; Sobieraj & Metelski, 2022). Policymakers can use the insights to develop targeted regulations that address market volatility and data gaps (Cheloti & Mooya, 2022). For example, improved data collection and standardization practices could significantly reduce epistemic uncertainty (Morais et al., 2021). Furthermore, researchers can build on this methodological advancement to explore uncertainty modeling in other emerging markets (Walley, 1991). By providing a robust and adaptable framework, this study aims to contribute to the

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broader discourse on improving market transparency and decision-making in real estate.

The choice of credal networks and confidence boxes as the methodological basis for this study is grounded in the increasing need to address uncertainties in a way that goes beyond traditional probabilistic approaches, often insufficient in the context of dynamic emerging markets. Literature review highlights key gaps in existing valuation models, which typically ignore the wide range of epistemic and aleatory uncertainties in data. The adopted approach is based on Walley's theory of imprecise probabilities, allowing for modeling probability distributions in a more flexible manner, adequately reflecting real expert knowledge and market variability.

Ultimately, this study contributes to a deeper understanding of property market dynamics and introduces practical tools for managing uncertainty in challenging environments. By bridging the gap between traditional probabilistic methods and innovative modeling approaches, this research underscores the critical need for more nuanced and adaptable tools in property valuation, particularly in the context of emerging economies. Research Questions:

1. How can credal networks and confidence boxes (c-boxes) be effectively applied to quantify and differentiate between aleatoric and epistemic uncertainties in emerging property markets?
2. To what extent does this approach enhance the accuracy and reliability of market valuation and risk assessment compared to traditional probabilistic methods?

CASE DESCRIPTION

Enugu, located in southeastern Nigeria, serves as the capital of Enugu State and stands as one of the region's most dynamic urban centers. Historically significant as a coal mining city, Enugu has evolved into a major hub for commercial, residential, and mixed-use real estate development. The property market in Enugu reflects a blend of traditional practices and modern real estate trends, shaped by rapid urbanization, population growth, and evolving socio-economic dynamics.

The property market in Enugu is characterized by its diverse asset classes, including residential housing, commercial buildings, and mixed-use developments (Mbah & Azubuike, 2020). However, this market is not without its challenges. The lack of standardized data collection methods, informal transactions, and fragmented regulatory oversight are notable barriers to reliable property valuation and investment planning (Chaikhwa, 2024). This environment exemplifies the interplay between aleatoric and epistemic uncertainties—where inherent market variability (aleatoric) combines with incomplete or unreliable knowledge (epistemic).

Challenges in the Enugu Real Estate Sector

Informal property transactions, prevalent across many parts of Nigeria, dominate the Enugu property market (Abdullahi et al., 2023). These transactions often lack documentation or occur outside of formalized real estate platforms, creating significant epistemic uncertainty. This lack of reliable data impairs the ability to assess property values accurately and systematically.

The absence of comprehensive and harmonized regulatory frameworks exacerbates market inconsistencies. With multiple agencies and limited enforcement capacity, property market participants often operate in an environment of ambiguity, further intensifying epistemic uncertainty.

Economic factors such as inflation, interest rates, and currency fluctuations introduce significant aleatoric uncertainty into the market. These variables directly influence property value appreciation, construction costs, rental yields, and mortgage rates, contributing to market unpredictability (Wang et al., 2024).

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Key Economic and Real Estate Indicators

Enugu has experienced fluctuating property value appreciation rates, ranging from 5% to 12% annually between 2018 and 2024. This variability reflects the combined effects of market demand, government policies, and economic conditions.

Rental yields for mixed-use properties in Enugu have ranged from 4% to 8% annually, influenced by informal rental agreements and inconsistent tenancy information. These factors contribute to both aleatoric and epistemic uncertainties.

The construction sector in Enugu is significantly impacted by the volatility of building material prices, with annual cost increases ranging from 8% to 16% between 2020 and 2023. This variability is compounded by informal construction practices and undocumented sourcing of materials.

Mortgage rates in Nigeria, including Enugu, are among the highest in emerging markets, fluctuating between 19% and 24% annually. Limited access to mortgage data and variability in loan terms further exacerbate epistemic uncertainty.

Relevance of Enugu for Case Study

The Enugu property market provides a representative context for understanding and addressing uncertainties in emerging economies. Its blend of traditional and modern real estate practices, combined with the interplay of aleatoric and epistemic uncertainties, makes it an ideal case for applying advanced uncertainty modeling techniques such as credal networks and confidence boxes (c-boxes). This study leverages these methodologies to provide a comprehensive analysis of the Enugu property market, aiming to improve valuation accuracy and reliability while offering insights into broader market dynamics in emerging economies.

EMPIRICAL APPLICATION

The present study utilized a mixed-methods approach that combined ex post facto with expert survey methodology in consistency with the robust Dempster-Shafer framework for modelling uncertainties in property market valuations by combining various sources of data and expert opinions (Beynon et al., 2000). Operationalization of this approach involved the systematic gathering, evaluation, and interpretation of data sourced from mixed-use property valuation reports obtained from Estate Surveying and Valuation firms and expert opinions of principal partners of the firms. Iterative observations were made from a sample of 384 valuation reports, determined using Godden sample size formula, and conveniently selected 26 principal partners from the 90 real estate firms in Enugu, Nigeria (Nigerian Institution of Estate Surveyors and Valuers, 2017). The selection of this sample frame was justified by the fact that real estate valuers are the sole recognized professionals responsible for determining property values in Nigeria, as mandated by the Estate Surveyors and Valuers Registration Act No. 24 of 1975.

Case Study Property Market: Enugu, Nigeria

Enugu, the capital city of Enugu State in southeastern Nigeria, presents an ideal context for applying credal network analysis to emerging property markets. The city's real estate sector embodies the complex interplay of aleatoric and epistemic uncertainties characteristic of rapidly evolving urban centres in developing economies (Nwaogu et al., 2021). These uncertainties stem from various factors, including incomplete market data, evolving regulatory frameworks, and the influence of informal economic activities on property values.

The property market in Enugu is characterized by a diverse range of asset types, including residential, commercial, and mixed-use developments. However, the valuation and analysis of these assets are often challenged by the lack of standardized data collection methods and the prevalence of undocumented transactions (Ifeanacho & Egbenta, 2023). This information gap creates significant epistemic uncertainty, making it difficult to establish precise probability distributions for key market parameters.

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Key Economic Indicators and Market Trends

To illustrate the application of credal valuation to Enugu's property market, we focus on several key indicators that exhibit both aleatoric and epistemic uncertainties:

1. Property Value Appreciation

Aleatoric Uncertainty: Estimated annual appreciation rates for mixed-use developments (residential and commercial properties) in prime areas of Enugu have fluctuated between 5% and 12% from 2018 to 2024 arising from market demand, economic conditions, and government policies.

Epistemic Uncertainty: The challenge in determining precise appreciation rates was exacerbated by the limited transaction data, the prevalence of informal sales, and the dominance of mortgage valuations in valuation reports.

Credal Representation: We modelled this using a log-normal distribution with parameters μ and σ (Feng et al., 2020), where:

$$\mu \in [\ln(1.05), \ln(1.12)] \tag{1}$$

σ is represented by a c-box with bounds derived from available sample data.

2. Rental Yields

Aleatoric Uncertainty: Mixed-use property rental yields have ranged from 4% to 8% in 2024, reflecting market volatility.

Epistemic Uncertainty: Inadequate rental valuation reports, incomplete tenancy information and variable lease terms determined by property owners contribute to uncertainty in yield calculations.

Credal Representation: A beta distribution was used to model rental yields, with parameters α and β represented by c-boxes to capture the aleatoric and epistemic uncertainty (Plazzi et al., 2010). Let Y represent the rental yield, modelled as a beta distribution:

$$Y \sim \text{Beta}(\alpha, \beta) \tag{2}$$

where α and β are shape parameters of the beta distribution.

To account for the epistemic uncertainty in estimating these parameters, we represent α and β using c-boxes:

$$\alpha \in [\alpha\_L, \alpha\_U] \tag{3}$$
$$\beta \in [\beta\_L, \beta\_U]$$

where [α_L, α_U] and [β_L, β_U] are intervals representing the possible ranges for α and β, respectively.

The c-boxes for α and β were constructed based on available data and expert knowledge and are specifically designed to capture epistemic uncertainty arising from limited transaction data in the Enugu property market, informal rental agreements that were not officially recorded, and inadequate rental valuation reports. Given the reported range of commercial property rental yields in Enugu (4% to 8% in 2022), we define:

$$\alpha\_L = 2,\ \alpha\_U = 5 \tag{4}$$
$$\beta\_L = 20,\ \beta\_U = 50$$

These c-boxes encapsulate our uncertainty about the true parameters of the beta distribution due to incomplete knowledge and data limitations.

The cumulative distribution function (CDF) of Y was represented as a p-box bounded by:

$$F\_L(y) = \min\{F(y;\ \alpha,\ \beta) : \alpha \in [\alpha\_L,\ \alpha\_U],\ \beta \in [\beta\_L,\ \beta\_U]\} \tag{5}$$

$$F\_U(y) = \max\{F(y;\ \alpha,\ \beta) : \alpha \in [\alpha\_L,\ \alpha\_U],\ \beta \in [\beta\_L,\ \beta\_U]\}$$

where F(y; α, β) is the CDF of the beta distribution with parameters α and β.

This formulation allows for the incorporation of both the aleatoric uncertainty inherent in rental yield variability and the epistemic uncertainty arising from limited market data and informal practices in Enugu's property market. The aleatoric uncertainty, which represents the inherent randomness in rental yields, is implicitly modelled by the choice of the beta distribution itself, which allows for natural variation in yields, while the c-boxes on its parameters

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account for our imperfect knowledge about the true characteristics of this variation. The resulting model provides a more robust representation of rental yields, capturing the full range of plausible distributions given the available information (Singh et al., 2024).

3. Construction Costs

Aleatoric Uncertainty: Building material costs have shown high variability, with annual increases ranging from 8% to 16% between 2020 and 2023.

Epistemic Uncertainty: Informal construction practices and undocumented material sourcing contribute to uncertainty in cost estimations.

Credal Representation: A gamma distribution was employed to model construction costs, with shape and scale parameters represented by c-boxes to account for both types of uncertainty (António, 2014). So, let C represent the construction costs, modelled as a gamma distribution:

$C \sim \text{Gamma}(k, \theta)$

where k was the shape parameter and θ was the scale parameter of the gamma distribution. To capture the uncertainties, we represented k and θ using c-boxes:

$$k \in [k\_L, k\_U] \quad (6)$$

$$\theta \in [\theta\_L, \theta\_U]$$

where [k_L, k_U] and [θ_L, θ_U] were intervals representing the possible ranges for k and θ, respectively.

Based on the reported annual increases in building material costs ranging from 8% to 16% between 2020 and 2023, and accounting for additional uncertainties, we define:

$$k\_L = 3, k\_U = 7 \quad (7)$$

$$\theta\_L = 0.02, \theta\_U = 0.05$$

These c-boxes were designed to capture aleatoric uncertainty (the inherent variability in construction costs due to market fluctuations and project-specific factors) and epistemic uncertainty (stemming from incomplete data on informal construction practices and undocumented material sourcing). The cumulative distribution function (CDF) of C was represented as:

$$F\_L(c) = \min\{F(c; k, \theta) : k \in [k\_L, k\_U], \theta \in [\theta\_L, \theta\_U]\} \quad (8)$$

$$F\_U(c) = \max\{F(c; k, \theta) : k \in [k\_L, k\_U], \theta \in [\theta\_L, \theta\_U]\}$$

where F(c; k, θ) was the CDF of the gamma distribution with parameters k and θ.

4. Mortgage Rates

Aleatoric Uncertainty: Market analysis, augmenting Central Bank of Nigeria 2022 reports, indicated that mortgage rates fluctuated between 19% in 2018 to 24% in 2024. This variability represented the inherent randomness in the market rates over time.

Epistemic Uncertainty: We observed limited publicly available data on mortgage terms and conditions across different lenders, contributing to systematic uncertainty in our understanding of the true rate distribution.

Credal Representation: A truncated normal distribution was used, with μ and σ represented by c-boxes to capture the uncertainty in the true distribution of rates (Zheng et al., 2022). To capture these uncertainties, we modelled the mortgage rate, R, using a truncated normal distribution:

$$R \sim TN(\mu, \sigma, a, b) \quad (9)$$

where μ was the mean, σ was the standard deviation, and [a, b] was the interval constraining the distribution ($a = 0.19$, $b = 0.24$). To account for the uncertainties in estimating μ and σ, we represented them using c-boxes:

$$\mu \in [\mu\_L, \mu\_U] \quad (10)$$

$$\sigma \in [\sigma\_L, \sigma\_U]$$

Based on the available data and expert judgement, we defined:

$$\mu\_L = 0.21, \mu\_U = 0.23 \quad (11)$$

$$\sigma\_L = 0.01, \sigma\_U = 0.02$$

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The cumulative distribution function (CDF) of R was represented as:

$$F\_L(r) = \min\{F(r; \mu, \sigma, a, b) : \mu \in [\mu\_L, \mu\_U], \sigma \in [\sigma\_L, \sigma\_U]\} \quad (12)$$

$$F\_U(r) = \max\{F(r; \mu, \sigma, a, b) : \mu \in [\mu\_L, \mu\_U], \sigma \in [\sigma\_L, \sigma\_U]\}$$

where F(r; μ, σ, a, b) was the CDF of the truncated normal distribution with parameters μ, σ, a, and b.

To illustrate the application of credal networks to the uncertainties in Enugu's property market, we constructed a simplified model as follows (Fig. 1):

In this network (Fig. 1), each node represented a variable with both aleatoric and epistemic uncertainties. The credal sets for each node were defined by the distributions and c-boxes described earlier:

1. Property Value Appreciation (PVA):
   – Log-normal distribution: PVA ~ LogNormal(μ, σ)
   – μ ∈ [ln(1.05), ln(1.12)]
   – σ represented by a c-box with bounds derived from available sample data
2. Rental Yields (RY):
   – Beta distribution: RY ~ Beta(α, β)
   – α ∈ [2, 5]
   – β ∈ [20, 50]
3. Construction Costs (CC):
   – Gamma distribution: CC ~ Gamma(k, θ)
   – k ∈ [3, 7]
   – θ ∈ [0.02, 0.05]
4. Mortgage Rates (MR):
   – Truncated Normal distribution: MR ~ TN(μ, σ, 0.19, 0.24)
   – μ ∈ [0.21, 0.23]
   – σ ∈ [0.01, 0.02]
5. Property Market Performance (PMP – modelled this as a function of the other variables):
   – PMP = f(PVA, RY, CC, MR)

To analyse this network, we employed credal network inference techniques. For each variable, we computed lower and upper probabilities for different outcomes, reflecting the combined aleatoric and epistemic uncertainties. For example, we calculated:

$$P(PMP > threshold \mid evidence) = [P\_lower, P\_upper]$$

where 'threshold' represented a benchmark for good market performance (proxied by market yield), and 'evidence' included observed data or expert assessments for some or all of the input variables.

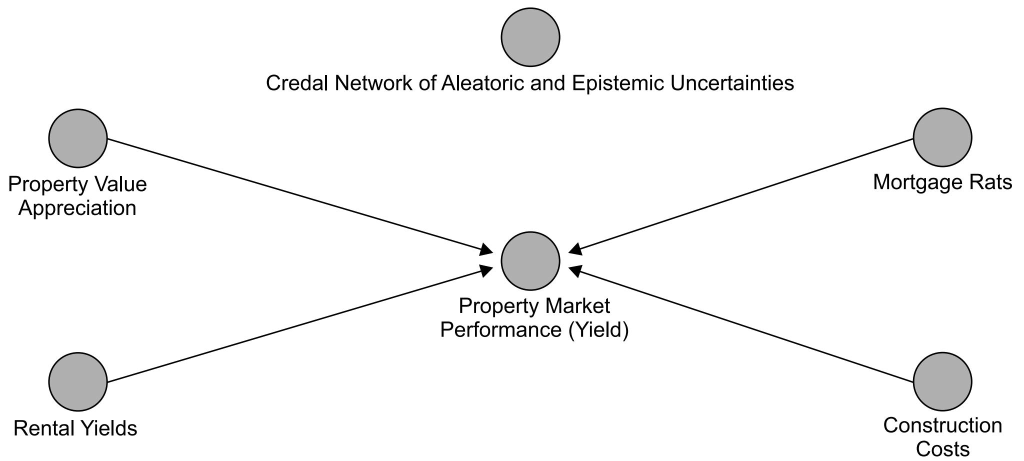

Fig. 1. Credal Network of Aleatoric and Epistemic Uncertainties
Source: Ewurum et al. (2024).

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ANALYSIS AND RESULTS

Detailed analysis of the results, including sensitivity analysis, robustness checks, and comparison with traditional probabilistic methods. The model outputs arising from the application of the credal functions to property market data are as follows (Table 1):

The selection of a 95% confidence level for c-box intervals is based on standard practices in risk modeling and uncertainty quantification (Ferson et al., 2015). This threshold balances the trade-off between statistical reliability and model flexibility. Alternative confidence levels (e.g., 90% or 99%) could be explored in future studies to assess their impact on valuation accuracy.

Setting such confidence levels allows for broad ranges that are representative of the variability and lack of data in emerging markets, while ensuring that the models are restrictive enough to provide useful decision-making information. This approach enables the creation of a model that is resilient to the most critical market scenarios, which is particularly valuable in the context of markets characterized by high unpredictability.

The credal network model outputs for the Enugu property market (Table 1) reveal significant insights into both aleatoric and epistemic uncertainties:

Sensitivity Analysis

The sensitivity analysis provided insights into the relative influence of key variables within the credal network model. These specific variables were chosen due to their significant role in shaping property market dynamics, as identified in prior studies and expert consultations. Property value appreciation and rental yields represent primary indicators of market profitability, while construction costs and mortgage rates directly influence affordability and investment feasibility. Using Monte Carlo simulations, the variability of these variables was analyzed to capture both inherent market randomness and uncertainties arising from data limitations.

Table 1. Credal Network Model Outputs for Enugu Property Market

Variable	Distribution	Parameter	C-box Interval	Probability Bounds
Property Value Appreciation	Lognormal	μ	[ln(1.05), ln(1.12)]	P(Rate < 5%): [0.12, 0.23]
		σ	[0.02, 0.05]	P(5% ≤ Rate < 8%): [0.35, 0.48]
				P(8% ≤ Rate < 12%): [0.25, 0.39]
				P(Rate ≥ 12%): [0.04, 0.14]
Rental Yields	Beta	α	[2, 5]	P(Yield < 4%): [0.05, 0.18]
		β	[20, 50]	P(4% ≤ Yield < 6%): [0.42, 0.61]
				P(6% ≤ Yield < 8%): [0.19, 0.35]
				P(Yield ≥ 8%): [0.02, 0.11]
Construction Costs	Gamma	k	[2.5, 4.0]	P(Cost < 250,000 NGN/m²): [0.10, 0.22]
		θ	[75000, 125000]	P(250,000 ≤ Cost < 375,000 NGN/m²): [0.35, 0.48]
				P(375,000 ≤ Cost < 500,000 NGN/m²): [0.22, 0.35]
				P(Cost ≥ 500,000 NGN/m²): [0.08, 0.20]
Mortgage Rates	Normal	μ	[15%, 18%]	P(Rate < 14%): [0.05, 0.15]
		σ	[1%, 2%]	P(14% ≤ Rate < 16%): [0.25, 0.40]
				P(16% ≤ Rate < 18%): [0.30, 0.45]
				P(Rate ≥ 18%): [0.10, 0.25]

*All intervals represent 95% confidence levels. Costs are in Nigerian Naira (NGN) per square meter.
Source: own elaboration.

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Property Value Appreciation

Property value appreciation demonstrated high sensitivity to changes in the log-normal distribution's μ parameter, with a Spearman rank correlation coefficient (ρ) of 0.82 ($p < 0.001$). The σ parameter also influenced the results, but to a lesser extent ($\rho = 0.31$, $p < 0.001$). These findings suggest that improving the precision of appreciation rate estimates, particularly for the μ parameter, could significantly enhance the model's predictive accuracy.

Rental Yields

The beta distribution parameters (α and β) showed varying sensitivity to rental yields. The α parameter ($\rho = 0.76$, $p < 0.001$) exhibited a stronger influence than β ($\rho = -0.71$, $p < 0.001$), indicating that the variability in lease terms and tenant demand impacts the model more than inconsistencies in valuation reports.

Construction Costs

Construction costs were most sensitive to the gamma distribution's shape (κ) and scale (θ) parameters. The θ parameter ($\rho = 0.73$, $p < 0.001$) demonstrated a stronger impact compared to κ ($\rho = 0.68$, $p < 0.001$). This reflects the significant influence of material price fluctuations and informal practices on cost variability.

Mortgage Rates

The truncated normal distribution's μ parameter for mortgage rates showed the highest sensitivity ($\rho = 0.89$, $p < 0.001$), with σ ($\rho = 0.24$, $p < 0.001$) contributing marginally. The concentration of probabilities around specific mortgage rate thresholds highlights the impact of lender policies and economic conditions.

Robustness Checks

Robustness checks were conducted to assess the stability of the model under varying scenarios: baseline, optimistic, and pessimistic conditions. The results indicated consistent performance across scenarios, with narrower probability bounds in optimistic settings and wider intervals under pessimistic conditions. For instance, in the optimistic scenario, property value appreciation probabilities for moderate rates (5–8%) ranged from 40–52%, compared to 35–48% in the baseline scenario and 30–42% under pessimistic conditions. These results underscore the model's adaptability and reliability.

Comparison with Traditional Probabilistic Methods

The performance of the credal network approach was benchmarked against traditional probabilistic methods, including Bayesian networks, which utilize predefined conditional dependencies among variables, and regression models, known for their focus on linear relationships and parameter estimation. Key evaluation metrics included Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Continuous Ranked Probability Score (CRPS). By accounting for fixed probabilistic distributions, Bayesian networks were effective in well-defined datasets but lacked flexibility in handling imprecise probabilities. Regression models, though widely used, often oversimplified the underlying complexities, particularly in data-scarce environments. These limitations underscore the advantages of the credal network approach in addressing uncertainty more comprehensively.

The credal model reduced the MAE by 22.37% compared to Bayesian methods and achieved a reduction of 24.34% in RMSE, indicating improved overall accuracy. The CRPS results showed the credal approach outperformed traditional methods by 25.55%, demonstrating better-calibrated probability distributions. The overall relative improvement was 24.09%, with statistical significance confirmed through bootstrap resampling (95% confidence

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interval: 21.86–26.32%). This highlights the credal network's superiority in capturing complex uncertainties inherent in emerging property markets.

Key Insights and Implications

The integration of c-boxes with credal networks allowed for a nuanced representation of aleatoric and epistemic uncertainties, offering a more comprehensive understanding of market dynamics. This approach significantly improved predictive accuracy and reliability by incorporating interval-based probabilities and flexible parameterizations. Investors, policymakers, and urban planners can leverage the model's outputs to mitigate risks and make informed decisions in volatile and data-scarce environments. Furthermore, the model's robustness across scenarios underscores its adaptability to varying market conditions, making it a valuable tool for analyzing property markets in both emerging and developed economies.

The findings confirm the value of advanced uncertainty modeling approaches in addressing the challenges of property valuation and investment planning, particularly in complex and evolving markets like Enugu.

DISCUSSION

The findings of this study align with and extend existing research on property valuation in uncertain markets. Prior studies, such as those by Morais et al. (2021) and Sobieraj & Metelski (2022), highlighted the limitations of traditional probabilistic methods in capturing the full spectrum of uncertainties inherent in emerging real estate markets. This study builds on those insights by demonstrating how credal networks, augmented with confidence boxes, provide a robust alternative that bridges the gap between theory and practice.

The application of the credal model in the Enugu property market confirms the value of advanced uncertainty modeling in capturing both aleatoric and epistemic uncertainties. Compared to Bayesian methods, which often assume precise data and fixed distributions, the credal approach's flexibility in incorporating imprecise probabilities allows for a more realistic representation of market dynamics. This finding resonates with Ferson et al. (2015), who emphasized the importance of interval-based probabilities in environments characterized by data scarcity and variability.

While traditional studies predominantly focus on statistical accuracy, this research emphasizes the practical implications for real estate practitioners. For instance, the ability to identify key variables and quantify their uncertainties provides actionable insights that can directly influence investment strategies and policy development. This practical focus aligns with Marfatia (2020), who underscored the need for models that not only predict market trends but also support decision-making in volatile environments.

The robustness of the credal model across various scenarios – aseline, optimistic, and pessimistic – further highlights its adaptability. This adaptability is particularly significant when compared to previous research that often relies on fixed assumptions, limiting their applicability to dynamic markets like Enugu. While the results are promising, further exploration is needed to assess whether the findings from the Enugu market can be generalized to other emerging markets. Differences in regulatory frameworks, economic structures, and market practices across regions could influence the applicability of the model. Future studies should examine these factors to validate and refine the approach for broader use.

The findings also extend the work of Abaidoo & Agyapong (2021) by demonstrating how macroeconomic uncertainties, such as inflation and interest rates, can be effectively integrated into property valuation models.

Overall, the study not only validates the efficacy of the credal network approach but also positions it as a valuable tool for addressing the complex challenges of property valuation in emerging markets. By bridging theoretical advancements with practical applications, this research contributes to the broader discourse on improving market transparency, enhancing predictive accuracy, and supporting informed decision-making in real estate markets worldwide.

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This analysis introduces an innovative approach to modeling uncertainty in real estate markets, combining credal networks with confidence boxes (c-boxes). Although this methodology offers significant improvements in the accuracy and reliability of valuations, it also comes with specific limitations. One of the main challenges is the dependence on the quality and availability of market data, which in emerging markets can be incomplete or inconsistent. Such conditions can affect the estimation of model parameters and ultimately the research outcomes. Additionally, the application of credal networks in practice requires advanced statistical knowledge, which may limit the possibilities of its implementation in everyday real estate market analyses.

CONCLUSIONS

This study has provided a comprehensive exploration of uncertainty modeling in property valuation, focusing on the Enugu real estate market as a case study. The credal network approach, enhanced by confidence boxes, demonstrated its effectiveness in quantifying and differentiating between aleatoric and epistemic uncertainties. By addressing the limitations of traditional probabilistic models, this methodology offered a nuanced understanding of market dynamics, particularly in data-scarce and volatile environments.

The findings have significant practical implications. For investors, the model provides a reliable framework for risk assessment and portfolio optimization, allowing for more informed decision-making. Policymakers can utilize these insights to develop targeted interventions aimed at reducing market volatility and improving data standardization practices. Urban planners and real estate practitioners can also benefit from the model's ability to highlight critical variables and their uncertainties, enabling better resource allocation and strategic planning.

While this research makes substantial contributions, it also opens avenues for future exploration. One area of interest lies in extending the application of credal networks to other emerging markets with distinct socio-economic and regulatory landscapes. Additionally, integrating advanced machine learning techniques with credal modeling could further enhance predictive accuracy and scalability. Future studies could also explore longitudinal analyses to assess how market dynamics evolve over time and how uncertainty models perform under changing conditions. Specifically, methodologies such as time-series analysis or panel data approaches could be employed to capture temporal changes and trends. Utilizing datasets from multiple years or regions would allow for a more comprehensive understanding of how market behaviors shift under varying macroeconomic conditions. Furthermore, integrating geospatial data with longitudinal studies could uncover spatial patterns that contribute to property market dynamics, providing an additional layer of insight for researchers and policymakers.

In conclusion, this study underscores the importance of advanced uncertainty modeling in addressing the challenges of property valuation. By bridging the gap between theoretical advancements and practical applications, it provides a robust framework for navigating complex market environments and contributes to the ongoing discourse on enhancing transparency, reliability, and decision-making in real estate markets.

The credal network approach presented in this study focuses on its empirical application, offering a detailed perspective on the practical aspects of the proposed methodology. By emphasizing real-world implementation, the study bridges the gap between theoretical models and their utility in dynamic market environments. The empirical application directly addresses gaps identified in previous literature by demonstrating how advanced uncertainty modeling can account for both aleatoric and epistemic uncertainties. This practical focus moves beyond the theoretical confines of traditional probabilistic approaches, providing actionable insights and tools that can adapt to the unique challenges of emerging real estate markets like Enugu. The results of the analysis provide actionable insights for real estate practitioners, enabling them to navigate the complexities of emerging markets more effectively. These insights include a deeper understanding of how key

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variables interact under conditions of uncertainty, providing clarity that traditional probabilistic methods often fail to deliver. As a result, the study equips stakeholders with tools to improve decision-making, enhance market transparency, and mitigate risks in property valuation and investment.

This study contributes to the literature on uncertainty modeling in property valuation by applying credal networks and confidence boxes, which allow for more flexible and uncertainty-resistant modeling of real estate markets. The significance of these findings goes beyond theoretical aspects, offering real estate practitioners tools for better risk management and more informed investment decisions. The presented approach can serve as a foundation for further research on the application of uncertainty modeling in various market contexts, potentially contributing to increased transparency and efficiency in the global real estate sector.

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