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MATHEMATICAL MODEL FOR ERRORS ESTIMATION OF DIFFERENCE OF OBJECTS'S ELEVATION DETERMINATION USING FLYING PLATFORM

Volodymyr Korolov[1], Krystyna Kurowska[✉2], Olha Korolova[3], Igor Milkovich[4], Yaroslav Zaiets[5]

[1] ORCID: 0000-0001-8421-584X
[2] ORCID: 0000-0002-1517-7402
[3] ORCID: 0000-0002-9265-8448
[4] ORCID: 0000-0003-4478-9536
[5] ORCID: 0000-0002-2697-3658
[1] Department Scientific Center of the Ground Forces,
Hetman Petro Sahaidachnyi National Army Academy, **Ukraine**
32 Heroes of Maidan street, Lviv, Ukraine, 79026
[2] Department of Spatial Analysis and Real Estate Market, University of Warmia and Mazury in Olsztyn
Prawocheńskiego street 15, 10-720 Olsztyn, **Poland**
[3] Department Scientific Center of the Ground Forces, Hetman Petro Sahaidachnyi National Army Academy, **Ukraine**
32 Heroes of Maidan street, Lviv, Ukraine, 79026
[4] Department of Electromechanics and Electronics, Hetman Petro Sahaidachnyi National Army Academy, **Ukraine**
32 Heroes of Maidan street, Lviv, Ukraine, 79026
[5] Department Scientific Center of the Ground Forces, Hetman Petro Sahaidachnyi National Army Academy, **Ukraine**
32 Heroes of Maidan street, Lviv, Ukraine, 79026

ABSTRACT

In this paper the method for determination of the object's elevation on the ground as well as the mathematical model for error estimation of object's determination are suggested. It is assumed that the object is situated beyond line-of-sight or, whatsoever, beyond the horizon.
It has been obtained the analytic expression for error estimation of the object's elevation determination using flying platform in case of the known inaccuracy of goniometry and distance measures.
The effect of inaccuracy of goniometry and distance measurements on the error estimation of the object's elevation determination is traced. Analytic expressions of evaluating requirements for accuracy of goniometer and distance measurer are deduced. These requirements will ensure the specified accuracy of the object's elevation determination.
A relationship between object's elevation determination and its location with respect to the observing station and flying platform is investigated.

Key words: elevation, object orientation, flying platform, error estimation

✉krystyna.kurowska@uwm.edu.pl

INTRODUCTION

In order to perform certain tasks, landmarks have to be identified in regions where existing optical systems cannot be used. Accurate determination of landmark parameters is largely dependent on the operational performance of measuring devices. Often the determination of the object's elevation on the ground is realized by the method of trigonometric leveling from local landmarks and reference point of the geodetic network (Ghilani et al. 2012, Whyte et al. 1997). The density of the reference point is lower in the mountains and wooded area. The equipment of the OS of the NS ensures its orientation and positioning. This makes the dependence of the possibility of observations on the availability of ground control points to disappear.

Often the objects, elevation of which on the ground to be determined, can be located beyond the line-of-sight range from the observing point. In some problems, they can be located beyond the horizon. For these reasons, the search for methods of elevation determination for such objects is still up-to-date/to the point. In this case as well as in the papers (Korolov et al. 2009a, Korolov et al. 2009b), it is proposed to use flying platform as additional observing station.

Apart from the navigation system, observing station is equipped with the devices that allow measuring of viewing angles of the objects and distances to them. The flying platform is equipped with both vertical signaling gyroscopic unit and devices, which allow measuring of viewing angles of the objects and distances to them. In the paper (Korolov et al. 2019) the problem of determining the object's plane coordinates with the use of flying platform is solved, but the issue of elevation determination remains open.

The purpose of this paper is to develop the method of object elevation determination without direct vision or in case its location beyond the horizon and its accuracy evaluation.

METHODOLOGY OF RESEARCH AND MATERIALS

To achieve this goal, we consider a geometric spatial model of the location of the flying platform relative to the observation station and the object. Math modeling was carried out using MatLab software.

To solve this problem the observing station (OS) is to be established, which is equipped with navigation system (NS). It provides object's orientation and positioning. Additionally, observing station is equipped with two devices: distance measuring instrument and position indicator. With respect to OS an airborne observing station is established. It is proposed to use flying platform for that purpose. This platform is equipped with vertical signaling gyroscopic unit and position indicator. The elevation of flying platform referred to OS is to be defined from the OS. By means of the abovementioned devices on flying platform, its elevation with respect to the object is determined. Their difference represents the elevation of the object.

Schematic illustration of mutual disposition of the flying platform referred to observing station and the object is shown in Figure 1.

The following symbols are used in the Figure 1:

K – the observing station;
H – the flying platform;
C – the object;
z_k – the observing station 's coordinate of elevation (above sea level);
$z_н$ – the flying platform 's coordinate of elevation;
z_c – the object's coordinate of elevation;
D_{KH} – the distance between observing station and flying platform;
D_{HC} – the distance between the flying platform and the object;
δ – the angle between the horizon and the direction from the observing station to the flying platform;
φ – the angle between the vertical passing through the flying platform and the direction from the flying platform to the object.

✉krystyna.kurowska@uwm.edu.pl

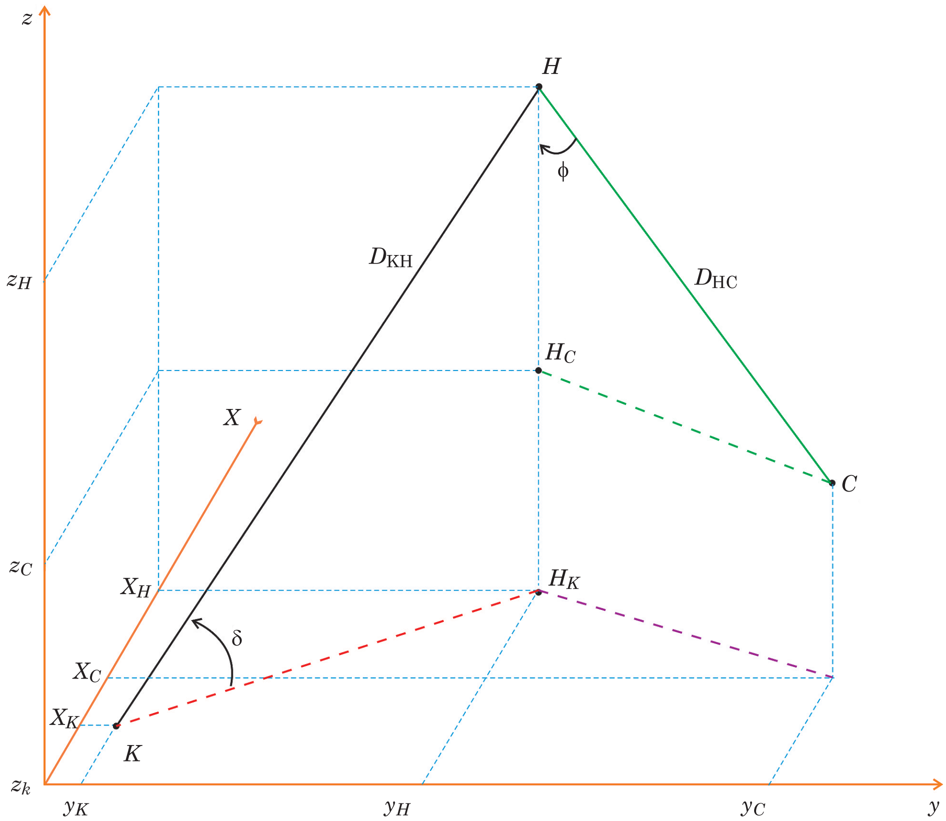

Fig. 1. The scheme of the mutual location of the observing station and object

Variable z_H – elevation of the flying platform H referred to observing station K is determined by trigonometric levelling (Ghilani et al. 2012)

$$z_H = D_{\mathrm{KH}} \cdot \sin \delta \tag{1}$$

Using similar approach, the elevation of the flying platform $\mathrm{HH_C}$ referred to the object C is found from the expression

$$\mathrm{HH_C} = D_{\mathrm{HC}} \cdot \cos \varphi \tag{2}$$

Geometrically, variable z_c – elevation of the object C, is found by using formula

$$z_c = z_k + D_{\mathrm{KH}} \cdot \sin \delta - D_{\mathrm{HC}} \cdot \cos \varphi \tag{3}$$

Function z_c is a multivariable function

$$z_c = z_c(z_k, D_{\mathrm{KH}}, \delta, D_{\mathrm{HC}}, \varphi) \tag{4}$$

The value of z_k is taken from the map, in the future its error is considered systematic. Its arguments D_{KH}, δ, D_{HC}, φ are the values of measurements of the respective distances and angles. With defined accuracy the measurements are taken by the assemblies, disposed at the observing station (D_{KH}, δ) and at the flying platform (D_{HC}, φ). Thus, the values of measured parameters D_{KH}, δ, D_{HC}, φ are random in character. So the function (4) can be considered as a function of multiple random variables.

In view of the foregoing, in order to estimate the error of object's elevation definition it is perfectly natural to use $\sigma^2_{z_c}$ function dispersion (4).

The fact that the parameters measurements D_{KH}, δ, D_{HC}, φ are taken by different operators, various devices, which are disposed in different locations, allows considering the errors of their measurements as statistically independent.

✉krystyna.kurowska@uwm.edu.pl

In this case for computing $\sigma^2_{z_c}$ we'll employ formula known from the probability theory (Ventcel 1969, Jeffreys 1998, Taylor 2015).

$$\sigma^2_{z_c} = \left(\frac{\partial z_c}{\partial D_{\mathrm{KH}}}\right)^2 \sigma^2_{D_{\mathrm{KH}}} + \left(\frac{\partial z_c}{\partial \delta}\right)^2 \sigma^2_{\delta} + \left(\frac{\partial z_c}{\partial D_{\mathrm{HC}}}\right)^2 \sigma^2_{D_{\mathrm{HC}}} + \left(\frac{\partial z_c}{\partial \varphi}\right)^2 \sigma^2_{\varphi} \quad (5)$$

Assuming that measurements of the linear and angular values both at the OS and flying platform are of equal accuracy: $D^2_{D\mathrm{KH}} = D^2_{D\mathrm{HC}} \equiv \sigma^2_D$, $D^2_{\delta} = D^2_{\varphi} \equiv \sigma^2_{\theta}$, and having performed appropriate manipulations, we obtain

$$\sigma^2_{z_c} = (\cos^2\varphi + \sin^2\delta)\sigma^2_D + \left(D^2_{\mathrm{HC}}\sin^2\varphi + D^2_{\mathrm{KH}}\cos^2\delta\right)\sigma^2_{\theta} \quad (6)$$

For a qualitative assessment of the relative effect of the additive components of the right-hand side of the equation (6) on the value of the variable $\sigma^2_{z_c}$ let us consider the diagrams of the behaviour in the same axes. Considerations will be carried out for the most typical configurations of the location of the OS, flying platform and the object.

For the quantitative error estimation of the object's elevation determination we will use the value of the variable σ_{z_c}.

DISCUSSIONS AND RESULTS

For studding the behaviour of the value $\sigma^2_{z_c}$ and its components, mathematical modeling with MatLab application was developed (Grant et al. 2008, Moore 2017). The example of the use of the angles and distances measuring devices was considered. These devices are used at the observing station and flying platform, they have such characteristics as maximum error in distance measurement is $\sigma_D = \pm 10$ m. Value σ_{θ} – is a maximum error in angles measurement by means of measuring device set up $\sigma_{\theta} = 2$ mrad (Korolov et al. 2000, Korolov et al. 2003, Bekir 2007). The abovementioned devices and their errors in measurement are taken as an example for clarity and quantitative assessment of the measurement of the necessary parameter in case of normal use. The results of the modeling are performed in Figure 2.

On the graphic plots (Fig. 2) for convenience of the results analysis, the values of $\sigma^2_{z_c}$, as well as the first and the second additive components of the formula (6) were marked along the vertical axis.

The calculation data are grouped into the first (*a*, *d*, *g*), the second (*b*, *e*, *h*) and the third (*c*, *f*, *i*) columns for the options, when KH_K (the distance along the flying platform horizon from the observing station) is 2000, 5000 and 7000 m, respectively.

The first (*a*, *b*, *c*), the second (*d*, *e*, *f*), and the third (*g*, *h*, *i*) rows contain the calculation data grouped for the options for flying platform disposal at an elevation of 5000, 2500, and 1000 m, respectively.

On the left horizontal axes we should mark the values of the flying platform elevation above the object from 0 to 5000, 2500, and 1000 m for the 1st, 2nd, and the 3rd column respectively.

On the right-hand horizontal axes we should plot the value of the horizontal distance from the object to the vertical, which passes through the flying platform from 0 to 5000 m for all options.

On the results of the analysis of the outlined graphic plots behavior it can be concluded that σ_{z_c} error in determining the elevation of the object varies for different options for placing the objects, which participate in remote sensing as follows:

- takes a value of up to 14 m at a elevation of the flying platform of about 5000 m;
- takes a value of up to 12–13 m at a elevation of a flying platform of about 2500 m;
- when the elevation of the flying platform is up to 1000 m, a zone of local minimum is observed – the value σ_{z_c} remains in the range of 5 to 8 m (at $\mathrm{KH}_K = 2000$ m and 7000 m, respectively).

From the graphic plots (*g*, *h*, *i*) it is apparent that σ_{z_c} takes the smallest value when $1500 \text{ m} < \mathrm{H}_C C < 3500$ m. When going beyond the range of the interval the variable σ_{z_c} increases, especially at small values $\mathrm{H}_C C$. Therefore the optimal positioning of the flying platform from the OS is at the elevations not less than

✉krystyna.kurowska@uwm.edu.pl

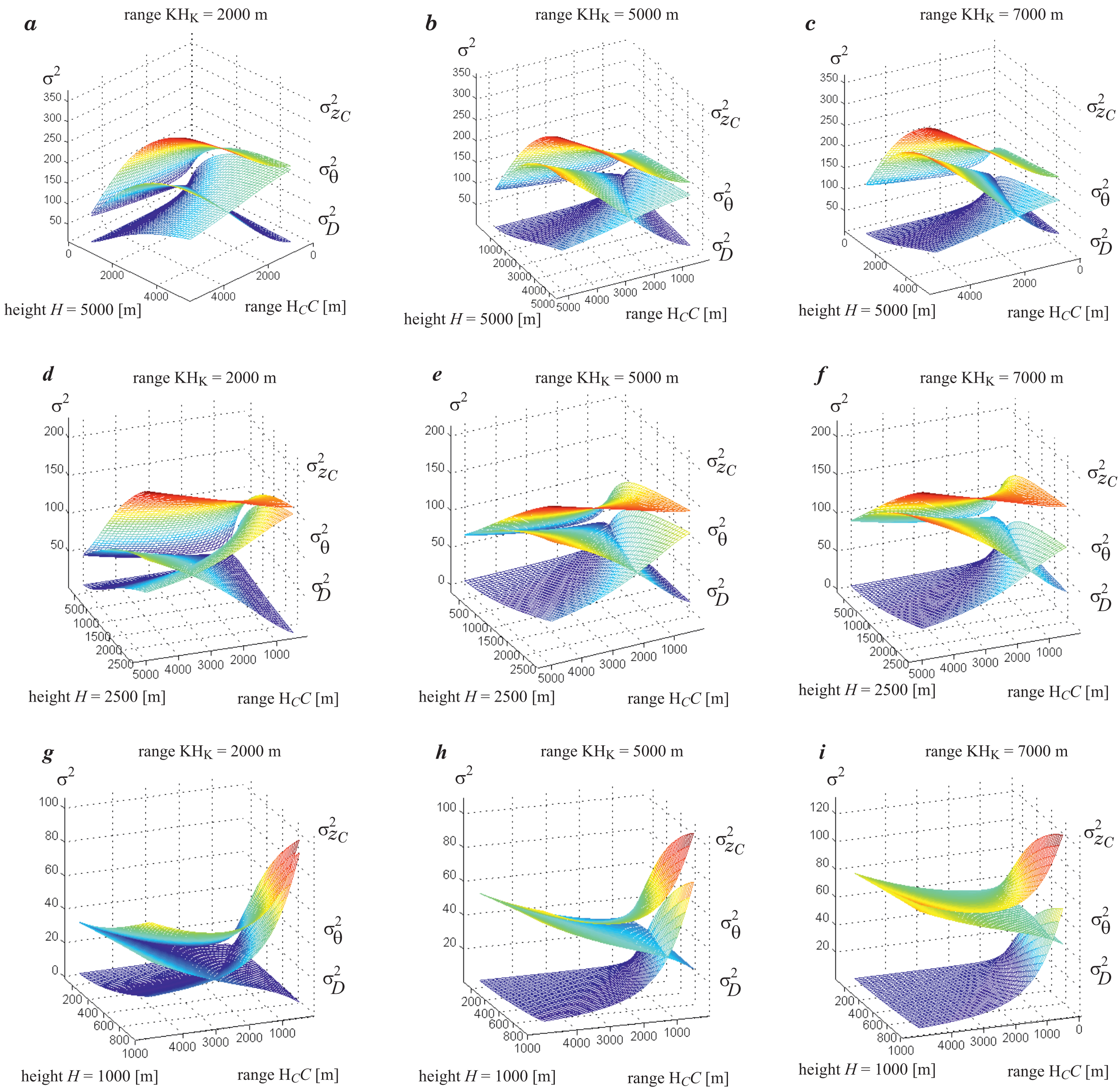

Fig. 2. Characteristics of the behavior of the value $\sigma^2_{z_c}$ and its components, depending on the location of the observing station, the flying platform and the object

1000 m and at the distance $H_C C$ from the ground control point when 1500 m < $H_C C$ < 3500 m.

For the values KH_K > 5000 m and $H_C C$ > 1500 m (the most typical regular situations) $\sigma^2_D(\cos^2\varphi + \sin^2\delta)$ the first element of the right-hand side of the formula (6) is by the order of magnitude lass than the second additive component. It allows ignoring it. Thus, we can conclude that the main component of the error in object's elevation determination σ_{z_c} is the error in angles measurement.

✉krystyna.kurowska@uwm.edu.pl

Then, to estimate the value σ_{z_c} we can apply the analytic dependence in the form:

$$\sigma_{z_c} = \sigma_\theta \cdot \sqrt{D_{\text{HC}}^2 \sin^2 \varphi + D_{\text{KH}}^2 \cos^2 \delta} \qquad (7)$$

Thus, with the known accuracy of the goniometers, formula (7) allows us to assess the potential accuracy of the object's elevation determination.

For the inverse problem. As may be required to provide with the specified accuracy of elevation determination, the formula

$$\sigma_\theta = \frac{\sigma_{z_c}}{\sqrt{D_{\text{HC}}^2 \sin^2 \varphi + D_{\text{KH}}^2 \cos^2 \delta}} \qquad (8)$$

allows to estimate the requirements for angular measurement accuracy.

Note, that in case of using of assemblies, providing geodetic accuracy of distances and angles measurement, the error in elevation determining will be correspondingly less, but the nature of its behavior will remain the same.

Further it is planned to develop the mathematical model for estimating the errors in determining the coordinates of the moving object, when they are determined with the use of the flying platform.

CONCLUSIONS AND PROPOSALS

1. It is shown that the error in angles measurement induces dominant component of the error in object's elevation measurement.

2. It is established that error in object's elevation measurement does not exceed 14 m, if the flying platform is set at the elevation no more than 5000 m (provided that the accuracy of measuring distances is not worse than ± 10 m and angles – not worse than 2 mrad). It is shown that the error in object's elevation determination varies from 5 to 8 m, if the flying platform elevation is not higher than 1000 m, the distance to the object is no more than 1500 m, distance to the observing station is no more than 3500 m.

3. Here is deduced the analytic expression for error estimation of the object's elevation determination, if the accuracy of the goniometers is known.

4. Here is obtained the analytic expression for assessing the requirements for angular measurement accuracy when determining the elevation of the object to the specified accuracy.

REFERENCES

Bekir, E. (2007). Introduction to modern navigation systems. World Scientific.

Ghilani C.D., Wolf P.R. (2012). Elementary surveying: an introduction to geomatics by Pearson Education, Inc., Upper Saddle River, New Jersey, 13th ed., p. 958.

Grant, M., Boyd, S., Ye, Y. (2008). CVX. Matlab software for disciplined convex programming.

Jeffreys, H. (1998). The theory of probability. OUP Oxford.

Korolov, V. (2000). Вимоги до характеристик навігаційної інформації і систем навігації наземних рухомих об'єктів в сучасному штатному процесі [Requirements for the characteristics of navigation information and navigation systems for land mobile objects in the modern staffing process]. Сучасні досягнення геодезичної науки та виробництва, 5, 280–283.

Korolov, V. 2003. Технічні вимоги до навігаційної інформації та сучасних систем навігації наземних рухомих об'єктів [Technical requirements for navigation information and modern navigation systems for land-based mobile objects]. Сучасні досягнення геодезичної науки та виробництва, 8, 218–221.

Korolov, V., Rudenko, K., Korolova, O. (2009a). Оцінка похибок визначення координат та вектора швидкості цілі з рухомого об'єкту [Estimation of errors of determination of coordinates and velocity vector of a target from a moving object]. VII Міжнародна науково-технічна конференція "Гіротехнології, навігація, керування рухом та конструювання авіаційно-космічної техніки", Issue 1, Київ: КПІ, pp. 210–215.

Korolov, V., Rudenko, K., Korolova, O. (2009b). Математична модель оцінки похибок визначення напрямку до рухомої цілі з нерухомого об'єкту координатним способом [Mathematical model of estimation of errors of determination of a direction to a mobile target from a fixed object by a co-

✉krystyna.kurowska@uwm.edu.pl

ordinate method]. Сучасні досягнення геодезичної науки та виробництва, 15(1), 232–236.

Korolov, V., Savchuk, S., Korolova, O., Milkovich, I., Zaec, Y. (2019). Mathematical model for errors estimation of object's location parameters determination using flying platform. Baltic surveying. International scientific journal (Latvia, Lithuania, Poland), 10, 26–30, doi: 10.22616/j.balticsurveying.2019.003.

Molinaro, A.M., Simon, R., Pfeiffer, R. M. (2005). Prediction error estimation. A comparison of resampling methods. Bioinformatics, 21(15), 3301–3307.

Moore, H. (2017). MATLAB for Engineers. Pearson.

Taylor J.R. (2015). An introduction to error analysis. The study of uncertainties in physical measurements. University Science Books Sausalito, California, p. 327.

Ventcel, E. (1969). Теория вероятностей [Probabilite theory]. Наука, Москва.

Whyte, W., Raymond R.E. (1997). Basic surveying, fourth edition 4th edition. Laxton's, 4 edition, p. 348.