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ASSESSING THE EFFICIENCY OF INVESTMENT FUND MANAGEMENT USING QUANTILE RISK MEASURES

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Abstract

The aim of the research is to compare the efficiency of managing selected Polish investment funds in various phases of stock market condition. The Value at Risk (VaR) and Conditional Value at Risk (CVaR) is used to construct efficiency ratios of fund management. Those funds investing in financial instruments have the most stable expected rate of return and the lowest risk, in all the analysed periods which made them highly effective.

The article also discusses the alternative methods to VaR and CVaR estimation which are used in the study. It is noted VaR and CVaR estimates obtained using backtesting and using APARCH models give similar results.

OCENY EFEKTYWNOŚCI ZARZĄDZANIA FUNDUSZAMI INWESTYCYJNYMI PRZY WYKORZYSTANIU KWANTYLOWYCH MIAR RYZYKA

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Słowa kluczowe: fundusz inwestycyjny, efektywność zarządzania funduszem, wartość zagrożona, warunkowa wartość zagrożona, model APARCH.

Abstrakt

Celem badań było porównanie skuteczności zarządzania wybranymi polskimi funduszami inwestycyjnymi w różnych fazach koniunkturalnych na rynku papierów wartościowych. Wartość zagrożona i warunkowa wartość zagrożona zostały wykorzystane do konstruowania wskaźników efektywności zarządzania funduszami. Fundusze inwestujące w instrumenty finansowe charakteryzowały się najbardziej stabilną oczekiwaną stopą zwrotu i najmniejszym ryzykiem we wszystkich analizowanych okresach, co przełożyło się na wysoką efektywność zarządzania tymi funduszami. Ponadto, w artykule zostały wykorzystane alternatywne metody estymacji VaR i CVaR. Zauważono, że zarówno dla VaR i CVaR oszacowanych metodą danych historycznych, jak i przy wykorzystaniu modeli APARCH, uzyskano podobne wyniki.

Introduction

Quantile measures of risk, like Value at Risk (*VaR*) are one of the widely-used risk measures due to the recommendation of the Basel Committee on Banking Supervision. This measure is also used in evaluating investment risk on the stock exchange. Quantile measures are used to analyse risk in emerging markets; the results of the research can be found in the paper of ATILGAN and DEMIRTAS (2013). Moreover, these measures function as the criterion of optimisation when choosing a securities portfolio (MAGHYEREH, AWARTANI 2012). Quantile measures are also used to assess investment risk on the commodity market (HAMMOUDEH et al. 2013). Also, relatively recently there were trials to use quantile measures to estimate risk at value of conditional volatility models. According to the authors of this approach, such models, despite their significant complexity, estimate *VaR* more precisely than the classic methods (e.g. GIOT, LAURENT 2003, ANGELIDIS et al. 2004, SO, YU 2006).

In order to judge the efficiency of managing investment funds a modified Campbell measure of efficiency is used (CAMPBELL et al. 2001). Efficiency ratios of investment portfolio management are used to compare investment portfolios with regard to profitability and risk. These measures evaluate a particular investment portfolio in term of expected profit and the degree of risk-free investments as well as the general situation in the market. A classic efficiency ratio of portfolio management is the Sharpe ratio, in which the excess of realised rate of return over the risk-free rate is referred to the risk measured with standard deviation (SHARPE 1966). Other popular efficiency ratios are based on the systematic risk of Jensen's Alpha (JENSEN 1968), Treynor's ratio (TREYNOR 1965) or M2 (MODIGLIANI 1997). In the case of the Polish capital market, an interesting analysis of the management effectiveness of the funds can be found in PIETRZYK'S article (2014). In his work 12 mutual fund were analysed. The topic of this research was market timing and the stock selection abilities of fund managers. A comprehensive analysis of the effectiveness of

investment fund management, yet excluding quantile measures, can be found in the work of ZAMOJSKA (2012).

The research aims to outline changes in the return of sales, risk and the management efficiency of particular fund types, depending on the situation in the financial markets. Further, various results obtained with different methods of *VaR* estimation were compared. The basic division of estimation methods came down to the use of backtesting and a method based on APARCH type of conditional volatility models. In the method of backtesting it is assumed that each observation has the same probability.

The aim of the research is to compare quantile risk and the efficiency of managing selected investment funds in various phases of stock market condition. It may be assumed that in a period of growth, aggressive funds are best-managed, whereas in a period of decline cash and bond funds are better.

Selected concepts of *VaR* and *CVaR*

Quantile measures can be estimated on the basis of the distribution function of certain theoretical distributions which are usually assumed as normal distributions. Other estimation methods are based on the method of backtesting, where the model of distribution is not taken into account. It is assumed that each observation in the past occurs with the same probability

In the classical approach, the value at risk (*VaR*) is understood as such a loss of market value of financial instrument that the probability of its occurrence, or exceeding in a planned period, equals the desired, close to zero tolerance level of h:

$$P(P_t \leq P_{t-1} - VaR) = \alpha, \tag{1}$$

where:
P_t – refers to instrument price in a moment t.

VaR may refer not only to the valuation level of a financial instrument in a given moment t, but also to the rate of return obtained in period t. Calculations in this article were carried out for rates of return, therefore, R_t denotes rate of return on a financial instrument. This rate can be treated as random variable, then *VaR* is α-quantile of its distribution, which is presented in the following equation:

$$P(R_t \leq - VaR) = \alpha. \tag{2}$$

Usually the left tail of distribution of return rates is analysed, assuming the probability on the level not higher than 0.1. As rates of return on financial instruments take on negative values in the bottom quantile, minus in the equation above means that estimation of VaR is usually obtained as positive value.

Another risk measure originating from *VaR* is Conditional Value at Risk (*CVaR*), which is defined as conditional expected value of return rates on a financial instrument provided that the rate of return takes on values lower than α-quantile. *CVaR*, corresponds with the medium level of loss, if the level of loss exceeds *VaR*, it can be written as:

$$CVar = E\{R_r \mid R_t \leq -Var(\alpha,R_t)\}, \tag{3}$$

where:
$Var(\alpha,R_t)$ – denotes value at risk for rate of return R_t and tolerance level α.

However, "classic" value at risk, as defined within equation (1) and (2) is not without faults. The most important fault involves the lack of fulfilling the condition of subadditivity, which means that *VaR* is not a coherent measure in the perspective of ARTZNER et al. (1999). In empirical studies (ROCKAFELLAR, URYASEV 2002) other faults of this risk measure were also pointed out. Conditional value at risk (*CVaR*) meets the condition of coherence and, therefore, in application (for instance, in portfolio analysis) this measure is more attractive than classic *VaR* (e.g. ROCKAFELLAR, URYASEV 2000, QUARANTA, ZAFFARONI 2008, LIM et al. 2011).

To estimate *VaR* calculations of conditional volatility were obtained with the use of conditional volatility models. This approach allows the inclusion of asymmetry in the conditional distributions of rates of return. As discussed above, this is to assure greater precision of *VaR* estimations. In this case, to determine the risk measure, estimations of volatility from the model and quantiles of adjusted distributions of innovation are used. Value at risk in period t is then estimated on the basis of conditional distributions of rates of return if provided with information available until moment $t - 1$, and is defined as:

$$P(R_t \leq -Var_t(\alpha,R_t) \mid \Omega_{t-1}) = \alpha, \tag{4}$$

where:
Ω_{t-1} – stands for the whole information available which determines the rate of return. In this article, to estimate *VaR* a model from the group of generalised models of autoregressive conditional heteroskedasticity APARCH (1, 1) (DING, GRANGER, ENGLE 1993) with the following specification:

$$Y_t = \sigma_t \varepsilon_t$$
$$\sigma_t^{\delta} = \varpi + \theta(|Y_{t-1}| - \gamma Y_{t-1})^{\delta} + \beta \sigma_{t-1}^{\delta}, \tag{5}$$

where:
$\delta > 0$, $-1 < \gamma < 1$, ϖ, θ, β – are model parameters.

In the aforementioned equations

$$Y_t = R_t - E(R_t \mid \Omega_{t-1}), \tag{6}$$

where:
$E(R_t \mid \Omega_{t-1})$ – is estimated with the use of models ARMA(n,m), represented as

$$E(R_t \mid \Omega_{t-1}) = \varphi + \sum_{i=1}^{n} \varphi_i R_{t-1} + \sum_{j=1}^{m} \eta_j Y_{t-j}, \tag{7}$$

and variables Y_t are IID with mean 0. Such action is justified as all analysed time series were stationary in the sense of standard ADF test of unit root occurrence. Symbol σ_t which occurs in equation (5) refers to conditional standard deviation, and ε_t is a sequence of independent variables with the same distribution of mean 0 and variance 1. In the case where the model parameters recognize certain specific values, the APARCH model (5) is reduced to one of the seven simple conditional variance models (see DING, GRANGER, ENGLE 1993). In the present, research it is assumed that ε_t can have normal distribution, Student's-t distribution or skew Student's-t distribution. On the basis of estimations from model (5) and (7), VaR forecast is estimated using the following formula:

$$VaR(\alpha,R_t) = -\mu_{t|t-1} - \sigma_{t|t-1} \cdot F_{\varepsilon}^{-1}(\alpha), \tag{8}$$

where:
$F_{\varepsilon}^{-1}(\alpha)$ – stands for α-quantile from a probability distribution of ε_t variable, whereas $\mu_{t|t-1}$ and $\sigma_{t|t-1}$ mean one-period forecasts of conditional expected value and volatility respectively.

To distinguish from risk measures represented by formula (2) and (3), quantile risk measures estimated on the basis of models of conditional volatility (5) and conditional expected value (7) are marked as $VaR_{\alpha}|\Omega_{t-1}$ and $CVaR_{\alpha}|\Omega_{t-1}$.

Campbell measure of risk and efficiency

A classic measure of portfolio management efficiency is the Sharpe ratio. It constitutes a premium for risk per unit of total risk, expressed with standard deviation:

$$Sh_S(p) = \frac{R_t(p) - R_f}{s_p}, \tag{9}$$

where:
$Sh_S(p)$ – the Sharpe ratio for portfolio p, R_t (p) – mean rate of return for portfolio p, R_f – mean risk-free rate of return, s_p – standard deviation of rate of return for portfolio p. The higher the value of this ratio, the better a particular investment fund is managed. Generally, the value of the Sharpe ratio for a particular investment portfolio is compared with the value of this ratio as a market index. If the value of the Sharpe ratio is higher than the value of the market index, it suggests that the fund is being managed efficiently.

Sortino modified the Sharpe ratio by replacing standard deviation with semi-deviation (SORTINO, SATCHEL 2001, p. 63). In a similar way, by using downside risk measures, one may modify other efficiency ratios for managing investment portfolio and obtain their downside counterparts (MISHRA, RAHMAN 2001, BACON 2008, p. 95–103).

CAMPBELL et al. (2001) in their study suggested VaR-based risk measure of portfolio investment. $W(0)$ stands for investment value in the initial moment of time and $VaR(\alpha,R_p)$ denotes VaR for rate of return on portfolio p. In the study, risk measure is used as:

$$\varphi(\alpha,p) = W(0) \cdot R_f - W(0) \cdot VaR(\alpha,R_p) \tag{10}$$

where:
R_f refers to risk-free rate available on the market. Bearing in mind the result obtained in (10), a counterpart of the Sharpe ratio is constructed:

$$Sh_\varphi(p) = \frac{R_t(p) - R_f}{\varphi(\alpha,p)}, \tag{11}$$

where:
R_t(p) – is the expected rate of return on investment in portfolio p (in the article, fund is identified with portfolio) in a moment of time t. Formula (11) informs of the expected profit from investment over the risk-free rate with regard to the

risk involved. As observed by CAMPBELL et al. (2001), in constructing the optimal portfolio maximizing ratio (11), the size of the initial capital does not have any impact on the choice of the optimal portfolio and, consequently, it does not influence evaluation of portfolio management efficiency performed with ratio (11). Therefore, in this article, a slightly simpler risk measure is proposed:

$$\varphi(\alpha,p) = R_f - VaR(\alpha,R_p), \tag{12}$$

with the same markings. Formula (12) was placed in formula (11) as denominator. Moreover, in this research, while estimating downside portfolio management efficiency ratios, *VaR* as well as *CVaR* were used in analogy to equation (12).

Empirical research

The quantile risk measures discussed in the article are estimated for fifteen selected investment funds managed by three investment fund companies operating in Poland. These funds have a varied declared levels of risk, and, as a result, a different structure of assets. Among them, are distinguish stock funds locating capital in the stocks of companies listed on foreign stock markets, including the Russian stock market; balanced funds, and also funds of financial instruments (including cash) and debt securities, which are commonly perceived as safe. Three of the most popular investment fund management companies in Poland are taken into account. The second criterion for selection is that the selected fund management companies administrated all types of investment funds. Instead of the original names of the funds we use a brief description of the funds. The full names of the funds are provided in Appendix 1. Simple daily returns were used and some statistics relating to them are provided in Appendix 1. To compute the statistics and results the authors used OxMetrixc 5 software.

To identify particular phases of the business cycle four market indices were taken into consideration: WIG, DAX, RTS, and NASDAQ. It should be stressed that two of them concern developed markets and the other two refer to emerging markets. The identification of certain phases has an arbitrary character and is carried out on the basis of ananalysis of the diagrams modeling the aforementioned indices. It is assumed that if the change of a trend is observed before it occurs in at least three diagrams, then the moment of its occurrence is accepted as the end of the observed phase of business cycle.

The study is conducted in various phases of the business cycle on financial markets. While analysing the time series of values of particular indices, four sub-periods are identified:

– I period (02.01.2006 – 28.12.2007) is the phase of strong growth in all analysed stock markets;

– II period (02.01.2008 – 27.02.2009) brought a crash in financial markets, which involved significant losses of value in the analysed indices;

– III period (02.03.2009 -10.08.2011) is the period of growing prices on all analysed financial markets;

– IV period (11.08.2011 – 25.03.2014) is another period of growth after a sudden and short-term collapse of prices in the financial markets. In this period, index values of WIG, DAX and NASDAQ were higher whereas RTS index had a slight risk-off trend. Characteristic for this sub-period was stagnation in the Russian financial market with concurrent growths on the Warsaw Stock Exchange, Frankfurt Stock Exchange and growing prices on over-the-counter stock exchange markets in the USA, Canada and Japan.

In the article, index POLONIA was assumed as the risk-free rate.

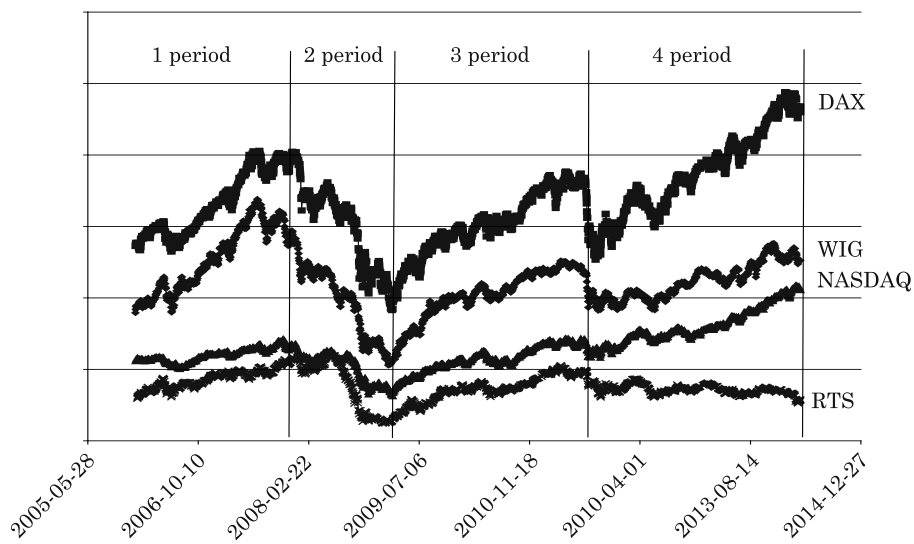

Fig. 1. Diagrams of indices and identified business cycle phases
Source: authors own elaboration.

In the study, there were problems connected with using APARCH models. In most cases, the model APARCH is reduced to GARCH. In addition, it is examined whether the use of the models IGARCH (ENGLE and BOLLERSLEV (1986)), EGARCH (BOLLERSLEV and MIKKELSEN (1996)) or GJR (GLOSTEN, JAGANNATHAN, RUNKLE (1993)) gives better results compared to the GARCH

model. All models are estimated three times, under different assumptions regarding the random component that is normal, *t*-student and skewed *t*-student. The final model to estimate VaR was chosen as the best model, meeting the assumptions of stationarity conditional variance and conditional average. However, it is impossible to estimate the parameters of any APARCH – GARCH models for some of the analysed funds (Appendix 2 summarises the best models used in the study). In the first analysed period (the phase of strong growth), risk based on the models of conditional volatility could not be estimated for 6 funds. The problems are numerical in nature and concerned funds with low declared level of risk (cash, bond, debt securities) as well as those with low quantile risk measured with *VaR* and *CVaR* estimated on the basis of backtesting, where we assume that each observation has the same probability. For each of the sub-periods, quantile risk measures presented above were estimated.

Further, efficiency ratios of fund management are calculated. Tables 1–4 present values of estimated risk measures, expected rate of return and estimated efficiency using ratio (12). All rates of return shown in the Tables are annualised rates of return on investments in a particular fund, calculated on the basis of daily values of rates of return.

In all the analysed periods, cash funds were characterised by the highest efficiency. In the periods of growth and stabilization, the most profitable were stock funds whereas in the period of decline, cash funds and debt securities, which is in accordance with expectations regarding investment funds.

In the period of a bull market, only one fund had negative efficiency, i.e. *PZU Papierów Dłuż. POLONEZ (PZU Government Bond)*. Its mean profitability is lower than the mean risk-free rate of return. It is surprising that in the period of strong price growth on financial markets, the highest efficiency characterised in cash and money funds which were also the least profitable. Nonetheless, they were very safe due to the fact that in at least 95% of cases their rates of return are positive.

The expected rate of return on stock funds is several times higher than the rate expected for cash funds. However, the risk associated with stock funds is also very high. The low values of efficiency ratio for stock funds indicate that the obtained rate of return is not sufficiently high for such a high level of risk.

In I period, due to numerical problems, it is impossible to estimate risk measures for APARCH conditional models of volatility. It is concerned with money funds and debt securities. It can be observed that in nearly all cases, estimations of *VaR* obtained with backtesting are lower than estimations obtained with APARCH models. As for *CVaR* estimations, the tendency is reversed, which is reflected in higher efficiency of *VaR* estimated with backtesting as compared with estimations from APARCH models. Efficiency estimations for *CVaR* are higher for models of conditional volatility.

Table 1
Fund risk and measures of management efficiency for $\alpha = 0.05$, 02.01.2006 – 28.12.2007 – bull market

Society/Investment fund	$E(R_p)$	VaR ($Sh_{VaR}(p)$*)	$VaR \mid \Omega_{t-1}$ ($Sh_{VaR}(p)$*)	$CVaR$ ($Sh_{CVaR}(p)$*)	$CVaR \mid \Omega_{t-1}$ ($Sh_{CVaR}(p)$*)
PZU Polish Equity	0.4356	0.9972 (0.0544)	0.9983 (0.0502)	0.9999 (0.0340)	0.9999 (0.0361)
PZU Government Bond	0.0379	0.3598 (-0.0084)	–	0.4672 (-0.0061)	–
PZU Money Market Fund	0.0574	0.0000 (0.3512)	–	0.1055 (0.0949)	–
PZU Stable Growth	0.1636	0.9192 (0.0433)	0.8985 (0.0475)	0.9785 (0.0285)	0.9738 (0.0301)
PZU Eastern European Equity	0.2066	0.9991 (0.0209)	0.9992 (0.0207)	0.9999 (0.0136)	0.9999 (0.0146)
Amplico Polish Equity	0.4743	0.9984 (0.0539)	0.9993 (0.0482)	0.9999 (0.0347)	0.9999 (0.0376)
Amplico Government Bond	0.0692	0.4370 (0.0417)	–	0.5121 (0.0338)	–
Amplico Money Market Fund	0.0559	0.0000 (0.3157)	–	0.2896 (0.0340)	–
Amplico Stable Growth	0.1526	0.9155 (0.0402)	0.9493 (0.0335)	0.9805 (0.0255)	0.9754 (0.0270)
Amplico Eastern European Equity	0.1766	0.9642 (0.0362)	0.9827 (0.0298)	0.9956 (0.0224)	0.9927 (0.0246)
Arka Polish Equity	0.5192	0.9994 (0.0515)	0.9984 (0.0585)	0.9999 (0.0340)	0.9999 (0.0361)
Arka Government Bond	0.0739	0.4376 (0.0486)	–	0.6532 (0.0273)	–
Arka Money Market Fund	0.0562	0.0000 (0.3235)	–	0.1782 (0.0563)	–
Arka Stable Growth	0.2138	0.9317 (0.0561)	0.8990 (0.0655)	0.9844 (0.0365)	0.9785 (0.0395)
Arka Eastern European Equity	0.3253	0.9983 (0.0378)	0.9990 (0.0350)	0.9999 (0.0216)	0.9999 (0.0210)

* Information in brackets reflects the modified Campbell measure of management efficiency
Source: authors own calculation.

Table 2
Fund risk and measures of management efficiency for $\alpha = 0.05$, 02.01.2008 – 27.02.2009 – bull market

Society/Investment fund	$E(R_p)$	VaR $(Sh_{VaR}(p)^*$	$VaR \mid \Omega_{t-1}$ $(Sh_{VaR}(p)^*$	$CVaR$ $(Sh_{CVaR}(p)^*$	$CVaR \mid \Omega_{t-1}$ $(Sh_{CVaR}(p)^*$
PZU Polish Equity	-0.6434	0.9999 (-0.0010)	0.9999 (-0.0799)	0.9999 (-0.0683)	0.9999 (-0.0780)
PZU Government Bond	0.0777	0.7072 (0.0002)	–	0.8873 (0.0095)	–
PZU Money Market Fund	0.0658	0.2902 (0.0003)	–	0.6853 (0.0082)	–
PZU Stable Growth	-0.2429	0.9797 (-0.0008)	0.9928 (-0.0670)	0.9965 (-0.0586)	0.9928 (-0.0671)
PZU Eastern European Equity	-0.5782	0.9999 (-0.0005)	0.9999 (-0.0609)	0.9999 (-0.0389)	0.9999 (-0.0454)
Amplico Polish Equity	-0.7065	0.9999 (-0.0010)	0.9999 (-0.0879)	0.9999 (-0.0735)	0.9999 (-0.0904)
Amplico Government Bond	0.0152	0.8169 (-0.0002)	0.9354 (-0.0139)	0.9693 (-0.0110)	0.9918 (-0.0080)
Amplico Money Market Fund	0.0576	0.0000 (0.0004)	0.0185 (0.0311)	0.4788 (0.0032)	0.4788 (0.0032)
Amplico Stable Growth	-0.3653	0.9927 (-0.0010)	0.9959 (-0.0923)	0.9995 (-0.0662)	0.9979 (-0.0825)
Amplico Eastern European Equity	-0.4509	0.9997 (-0.0008)	0.9999 (-0.0665)	0.9999 (-0.0478)	0.9999 (-0.0583)
Arka Polish Equity	-0.6770	0.9999 (-0.0009)	0.9999 (-0.1250)	0.9999 (-0.0659)	0.9999 (-0.0797)
Arka Government Bond	0.0689	0.5319 (0.0002)	0.7687 (0.0085)	0.7642 (0.0086)	0.9955 (0.0024)
Arka Money Market Fund	0.0150	0.5896 (-0.0004)	0.5455 (-0.0462)	0.7765 (-0.0251)	0.9448 (-0.0132)
Arka Stable Growth	-0.3393	0.9885 (-0.0010)	0.9775 (-0.1223)	0.9996 (-0.0601)	0.9981 (-0.0746)
Arka Eastern European Equity	-0.7008	0.9999 (-0.0008)	0.9999 (-0.1019)	0.9999 (-0.0555)	0.9999 (-0.0615)

* Information in brackets reflects the modified Campbell measure of management efficiency
Source: authors own calculation.

In the period of the market crash, only bond funds and cash funds don't incur losses. However, for two of them, i.e. *Arka BZ WBK Ochrony Kapitału (Arka Money Market Fund)* and *Amplico Obligacji Skarbowych kat. A (Amplico Government Bond)*, the efficiency ratio is negative. In the case of all stock funds with probability 0,05, investors lost nearly 100% ($VaR_{0.05} = 0.9999$) of the invested capital per year with an average loss level from 58% to 70% per annum ($CVaR_{0.05}$).

In II period, similar to period I, it is impossible to estimate *VaR* and *CVaR* values using APARCH models for all the analysed funds.

In the period of the bear market, efficiency of funds is evidently lower than in the periods of growth and stabilisation. In the period of the stock market crash, it was only the bond and cash funds that didn't incur any losses. Similarlyin the bull market period, in decline period estimations of *VaR* obtained with models of conditional volatility are higher than estimations obtained with backtesting. Yet, for *CVaR* they are lower.

Table 3

Fund risk and measures of management efficiency for $\alpha = 0.05$, 02.03.2009 – 10.08.2011 – bull market

Society/Investment fund	$E(R_p)$	VaR $(Sh_{VaR}(p)^*$	$VaR \mid \Omega_{t-1}$ $(Sh_{VaR}(p)^*$	$CVaR$ $(Sh_{CVaR}(p)^*$	$CVaR \mid \Omega_{t-1}$ $(Sh_{CVaR}(p)^*$
PZU Polish Equity	0.2752	0.9984 (0.0331)	0.9999 (0.0132)	0.9999 (0.0225)	0.9998 (0.0257)
PZU Government Bond	0.0918	0.6108 (0.0595)	0.4162 (0.1019)	0.7812 (0.0374)	0.7438 (0.0417)
PZU Money Market Fund	0.0719	0.1586 (0.1945)	0.0804 (0.3462)	0.2811 (0.1095)	0.2752 (0.1121)
PZU Stable Growth	0.1145	0.9075 (0.0326)	0.9953 (0.0147)	0.9675 (0.0228)	0.9579 (0.0246)
PZU Eastern European Equity	0.3576	0.9996 (0.0351)	0.9999 (0.0177)	1.0000 (0.0259)	0.9999 (0.0273)
Amplico Polish Equity	0.3072	0.9988 (0.0357)	0.9999 (0.0170)	0.9999 (0.0248)	0.9998 (0.0277)
Amplico Government Bond	0.1637	0.7455 (0.0871)	–	0.9112 (0.0498)	–
Amplico Money Market Fund	0.0977	0.2338 (0.2134)	0.1177 (0.4072)	0.4046 (0.1153)	0.3866 (0.1219)
Amplico Stable Growth	0.1583	0.9208 (0.0457)	0.9938 (0.0231)	0.9803 (0.0297)	0.9810 (0.0295)
Amplico Eastern European Equity	0.2573	0.9914 (0.0418)	0.9997 (0.0249)	0.9983 (0.0312)	0.9974 (0.0336)
Arka Polish Equity	0.2732	0.9983 (0.0335)	0.9999 (0.0118)	0.9999 (0.0219)	0.9997 (0.0259)
Arka Government Bond	0.0989	0.2480 (0.2043)	0.3526 (0.1385)	0.4387 (0.1060)	0.4697 (0.0969)
Arka Money Market Fund	0.0867	0.1455 (0.2837)	0.1991 (0.2108)	0.3639 (0.1102)	0.4345 (0.0886)
Arka Stable Growth	0.1745	0.9067 (0.0546)	0.9968 (0.0228)	0.9730 (0.0361)	0.9564 (0.0415)
Arka Eastern European Equity	0.4260	0.9991 (0.0469)	0.9999 (0.0172)	0.9999 (0.0306)	0.9999 (0.0349)

* Information in brackets reflects the modified Campbell measure of management efficiency
Source: authors own calculation.

In III period, all funds were characterized by positive return and positive efficiency. Nonetheless, in this period the most efficient are cash funds while the least effective are stock funds.

In III period, alike in period I, the most profitable are stock funds. However, one may observe differences in profitability of particular types of stock funds. In period I, the highest rates of return are on funds investing in stocks on developed markets and in Poland, whereas in period III on those funds which invested in the markets of East and Middle East Europe.

In period I and II, estimations of *VaR* and *CVaR* with both methods were generally similar. In III period, there were significant discrepancies in the estimations made with various methods of risk measure for the estimated risk

Table 4

Fund risk and measures of management efficiency for $\alpha = 0.05$, 11.08.2011 – 25.03.2014 – phase of further growth on certain part of markets after a short-term crash

Society/Investment fund	$E(R_p)$	VaR ($Sh_{VaR}(p)$*)	$VaR \mid \Omega_{t-1}$ ($Sh_{VaR}(p)$*	$CVaR$ ($Sh_{CVaR}(p)$*	$CVaR \mid \Omega_{t-1}$ ($Sh_{CVaR}(p)$*
PZU Polish Equity	0.1672	0.9974 (0.0200)	0.9997 (0.0145)	0.9999 (0.0123)	0.9998 (0.0139)
PZU Government Bond	0.1255	0.6370 (0.0782)	0.6663 (0.0724)	0.7749 (0.0538)	0.8280 (0.0457)
PZU Money Market Fund	0.0676	0.1783 (0.1253)	0.1674 (0.1329)	0.2721 (0.0824)	0.2725 (0.0823)
PZU Stable Growth	0.1118	0.9236 (0.0268)	0.9755 (0.0187)	0.9788 (0.0180)	0.9732 (0.0192)
PZU Eastern European Equity	0.1237	0.9975 (0.0134)	0.9987 (0.0121)	0.9999 (0.0091)	0.9998 (0.0097)
Amplico Polish Equity	0.1059	0.9978 (0.0106)	0.9997 (0.0080)	0.9999 (0.0068)	0.9999 (0.0072)
Amplico Government Bond	0.0712	0.5491 (0.0390)	0.4349 (0.0535)	0.7358 (0.0238)	0.7579 (0.0224)
Amplico Money Market Fund	0.0719	0.2193 (0.1167)	0.1508 (0.1658)	0.4242 (0.0563)	0.3378 (0.0739)
Amplico Stable Growth	0.0877	0.8698 (0.0231)	0.9163 (0.0191)	0.9703 (0.0135)	0.9675 (0.0139)
Amplico Eastern European Equity	-0.0158	0.9794 (-0.0134)	0.9901 (-0.0113)	0.9966 (-0.0092)	0.9963 (-0.0093)
Arka Polish Equity	0.1053	0.9950 (0.0121)	0.9990 (0.0093)	0.9998 (0.0073)	0.9998 (0.0076)
Arka Government Bond	0.0867	0.6051 (0.0486)	0.8251 (0.0264)	0.7955 (0.0289)	0.9817 (0.0117)
Arka Money Market Fund	0.0545	0.1224 (0.1007)	0.1686 (0.0760)	0.2390 (0.0543)	0.3613 (0.0347)
Arka Stable Growth	0.0995	0.8757 (0.0277)	0.8873 (0.0265)	0.9630 (0.0176)	0.9629 (0.0177)
Arka Eastern European Equity	0.0569	0.9987 (0.0029)	0.9978 (0.0031)	0.9999 (0.0020)	0.9999 (0.0021)

* Information in brackets reflects the modified Campbell measure of management efficiency
Source: authors own calculation.

with *VaR*. It is particularly noticeable for stock funds, where *VaR* estimations conducted with APARCH models are often over 10 pts higher than estimations conducted with backtesting. Concurrently, for *CVaR* these discrepancies are insignificant.

In the final studied period (Tab. 4), there are small discrepancies between profitability of particular funds. The return of stock funds slightly outbalances the return of the remaining types of funds. In this period, *PZU Papierów Dłużnych POLONEZ (PZU Government Bond)* has a similar return, in some cases even higher when compared with stock funds. Concurrently, *VaR* and *CVaR* values estimated with backtesting in period I and IV produced similar results. *Amplico Zrówna. Nowa Europa* is a very interesting case as it is the only fund which in this period has a mean negative return and negative efficiency ratio.

The ranking of funds with respect to estimated risk and management efficiency, performed on the basis of selected risk measures, is similar. The only change is the position of funds within a group of funds with similar risk level. To evaluate the degree to which the rankings of funds (according to risk measures and efficiency ratios) were similar, correlation coefficients between estimations for the analysed funds were determined (Tab. 5).

Table 5
Correlation coefficients between selected quantile risk measures and efficiency ratios of fund management based on them

Period		Correlation of risk measures				Correlation of efficiency measures			
		VaR	*CVaR*	*VaR* | Ω	*CVaR* | Ω	*VaR*	*CVaR*	*VaR* | Ω	*CVaR* | Ω
First period (bull market)	*VaR*	1.000	0.997	0.964	0.973	1.000	0.775	0.915	0.977
	CVaR	0.997	1.000	0.973	0.994	0.775	1.000	0.937	0.995
	VaR | Ω	0.964	0.973	1.000	0.958	0.915	0.937	1.000	0.923
	CVaR | Ω	0.973	0.994	0.958	1.000	0.977	0.995	0.923	1.000
Second period (Bear market)	*VaR*	1	0.996	0.940	0.982	1	0.977	0.931	0.948
	CVaR	0.996	1	0.949	0.982	0.977	1	0.917	0.985
	VaR | Ω	0.940	0.949	1	0.925	0.931	0.917	1	0.901
	CVaR | Ω	0.982	0.982	0.925	1	0.948	0.985	0.901	1
Third period	*VaR*	1	0.997	0.970	0.998	1	0.974	0.835	0.923
	CVaR	0.997	1	0.985	0.998	0.974	1	0.904	0.984
	VaR | Ω	0.970	0.985	1	0.973	0.835	0.904	1	0.948
	CVaR | Ω	0.998	0.998	0.973	1	0.923	0.984	0.948	1
Fourth period	*VaR*	1	0.997	0.973	0.979	1	0.982	0.948	0.946
	CVaR	0.997	1	0.983	0.983	0.982	1	0.905	0.944
	VaR | Ω	0.973	0.983	1	0.973	0.948	0.905	1	0.968
	CVaR | Ω	0.979	0.983	0.973	1	0.946	0.944	0.968	1

Source: authors own calculation.

Estimations of *VaR* are varied, depending on the assumed definition and method of estimation. On the basis of the results it can be concluded that the ranking of funds due to risk is similar with respect to all variants of *VaR* taken into consideration in the present study, which is confirmed by values of Pearson's linear correlation coefficients shown in Table 5.

All values of these coefficients are positive and statistically significant ($\alpha = 0.05$). As a result, it may be assumed that the analysed investment funds are characterised by a similar model of bottom quantile of rates of return distribution, which can be deduced from the strong correlation between *VaR* and *CVaR*.

Conclusions

In all the analysed periods, money funds have the highest efficiency. In the periods of growth and stabilization, the most profitable are stock funds, whereas in the period of decline, cash and debt securities funds offered the highest profitability, which is in tune with the expectations regarding investment funds. The ranking of funds based on particular risk measures is similar.

As expected, funds investing in financial instruments are characterised by a more stable expected rate of return and the lowest risk in all the analysed periods. Contrary to expectations, stock funds in the periods of a bull market on capital markets, despite high expected rates of return, are not ranked high with respect to fund management efficiency. It resulted from the high level of investment risk in those funds.

It is typical of balanced and stable growth funds to regularly obtain low management efficiency evaluations. In II period, low efficiency evaluation also characterised stock funds, which is connected with significant losses in value of these funds. In IV period, the lowest efficiency had funds locating capital in securities listed on Eastern markets, which is related to low rates of return on investments which do not involve lower investment risk.

Risk in IV period is similar to risk in I period. Investment portfolios in funds are similarly diversified with respect to risk, which may imply a stable investment policy of funds. Concurrently, profitability of funds became alike and the differences between stock funds and other funds became insignificant. This also concerned efficiency ratio of fund management.

An interesting conclusion can be drawn with regard to the *VaR* estimation method. If *VaR* is estimated in a particular moment of time (with backtesting), it is impossible to acknowledge the advantage of econometric models over backtesting. Estimations of risk measures obtained with two methods for various market situations were similar. As estimation of risk changes applied

consistently for time series is of low value for backtesting, in this aspect econometric models gain advantage. On the other hand, estimation with APARCH models was at times impossible, which was of key importance in this study as the goal was to evaluate fund management efficiency in a particular situation on the financial markets. This certainly is a disadvantage which was of less importance to current financial data. Nevertheless, in the case of emerging markets, APARCH models as tools to estimate *VaR* are not always recommended due to the quality of the available data and frequency of their sampling. As for exchange quotations characterised by long periods of unchanged rates of return (revaluation of fund unit value is carried out every few days, but quotations are published on a daily basis), the natural volatility of time series becomes distorted. As a result, models based on conditional volatility do not allow parameter estimators of sufficient value to be obtained. In the case of the backtesting method, which is based on the frequency of events, disturbances in the volatility of a studied period do not have such a big impact on quantile estimation of risk measure. Therefore, in this study, a simpler and definitely equally effective approach is to apply backtesting to evaluate risk and efficiency of investment funds in a specified market situation.

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APPENDIX 1

Original full names of funds

Society/Investment fund	Full name
PZU Polish Equity	PZU Akcji KRAKOWIAK *http:// www.pzu.pl/produkty/pzu-akcji-krakowiak*
PZU Government Bond	PZU Papierów Dłużnych POLONEZ *https://www.pzu.pl/produkty/pzu-papierow-dluznych-polonez*
PZU Money Market Fund	PZU Gotówkowy *https://www.pzu.pl/produkty/pzu-fio-gotowkowy*
PZU Stable Growth	PZU Stabilnego Wzrostu MAZUREK *https://www.pzu.pl/produkty/pzu-stabilnego-wzrostu-mazurek*
PZU Eastern European Equity	PZU Akcji NOWA EUROPA *https://www.pzu.pl/produkty/pzu-akcji-nowa-europa*
Amplico Polish Equity	Amplico Akcji Polskich kat. A *http://notowania.metlifefundusze.pl/index.php*
Amplico Government Bond	Amplico Obligacji Skarbowych *http://notowania.metlifefundusze.pl/index.php*
Amplico Money Market Fund	Amplico Pieniężny *http://notowania.metlifefundusze.pl/index.php*
Amplico Stable Growth	Amplico Stabilnego Wzrostu *http://notowania.metlifefundusze.pl/index.php*
Amplico Eastern European Equity	Amplico Zrównoważony NOWA EUROPA *http://notowania.metlifefundusze.pl/index.php*
Arka Polish Equity	Arka FIO Akcji A *http://arka.pl/produkty/produkty.html*
Arka Government Bond	Arka Obligacji skarbowych A *http://arka.pl/produkty/produkty.html*
Arka Money Market Fund	Arka Ochrony Kapitału (pieniężny) A *http://arka.pl/produkty/produkty.html*
Arka Stable Growth	Arka FIO Stabilnego Wzrostu A *http://arka.pl/produkty/produkty.html*
Arka Eastern European Equity	Arka FIO Akcji Środkowej i Wschodniej Europy (PLN) A *http://arka.pl/produkty/produkty.html*

Statistics of funds returns; 2.01.2006 – 28.12.2007 – bull market

Society/Investment fund	Standard deviation	Skewness	Excess Kurtosis	JB normality test
PZU Polish Equity	0.010892	-0.57601	1.617164	81.96851
PZU Government Bond	0.000794	-0.01036	2.336554	113.5207
PZU Money Market Fund	0.000134	0.322998	1.329768	45.44215
PZU Stable Growth	0.004405	-0.54568	1.531213	73.51287
PZU Eastern European Equity	0.012281	-0.67231	2.85688	207.288
Amplico Polish Equity	0.011453	-0.65728	1.973821	116.9332
Amplico Government Bond	0.00107	0.839261	4.118223	411.2009
Amplico Money Market Fund	0.00038	2.12188	6.395649	1224.917
Amplico Stable Growth	0.004448	-0.73401	2.499144	174.6663
Amplico Eastern European Equity	0.006262	-0.57322	2.389911	146.0824
Arka Polish Equity	0.012532	-0.79587	2.958991	234.7221
Arka Government Bond	0.001304	0.435905	4.159236	375.483
Arka Money Market Fund	0.000294	0.295023	2.075343	96.78939
Arka Stable Growth	0.004719	-0.8144	3.096009	254.4534
Arka Eastern European Equity	0.012889	-1.03262	6.789184	526.6631
POLONIA	0.004418	-0.06616	2.989743	186.2117

Statistics of funds returns; 2.01.2008 – 27.02.2009 – bear market

Society/Investment fund	Standard deviation	Skewness	Excess Kurtosis	JB normality test
PZU Polish Equity	0.017581	-0.22145	1.423464	26.94665
PZU Government Bond	0.002425	-0.9081	7.607309	741.6835
PZU Money Market Fund	0.001457	-0.58993	46.14083	25830.72
PZU Stable Growth	0.00626	-0.45621	2.230781	70.43287
PZU Eastern European Equity	0.026377	0.176919	4.158021	211.1489
Amplico Polish Equity	0.019097	-0.22417	1.826356	42.88109
Amplico Government Bond	0.003755	-3.01004	35.92034	16083.96
Amplico Money Market Fund	0.000778	-5.48595	61.21188	46890.74
Amplico Stable Growth	0.007701	-0.55624	3.443826	158.808
Amplico Eastern European Equity	0.015098	-0.12427	4.824209	282.934
Arka Polish Equity	0.019853	-0.09299	2.133664	55.61877
Arka Government Bond	0.00173	-1.8859	21.20852	5626.335
Arka Money Market Fund	0.001376	-1.84714	10.67431	1547.012
Arka Stable Growth	0.007973	-0.48113	3.549698	164.0065
Arka Eastern European Equity	0.025646	0.226294	3.359008	139.2892
POLONIA	0.007595	-1.12504	1.490734	88.33273

Statistics of funds returns; 2.03.2009 – 10.08.2011 – growth after decline

Society/Investment fund	Standard deviation	Skewness	Excess Kurtosis	JB normality test
PZU Polish Equity	0.011586171	0.053317942	2.315269083	136.3104943
PZU Government Bond	0.0017628	0.141616074	7.196760902	1316.292294
PZU Money Market Fund	0.000507254	-1.81074327	35.95365716	33134.18343
PZU Stable Growth	0.004113361	-0.095087677	1.856562619	88.38090739
PZU Eastern European Equity	0.013168725	-0.154495828	1.519724195	61.0278259
Amplico Polish Equity	0.011861006	0.125644956	2.65352879	180.2731772
Amplico Government Bond	0.003380377	3.774823566	58.22959762	87484.96148
Amplico Money Market Fund	0.000794307	0.868468235	12.15143952	3823.363679
Amplico Stable Growth	0.004818685	0.225349143	3.856702014	382.585964
Amplico Eastern European Equity	0.008340712	0.337994436	2.298194476	145.6184672
Arka Polish Equity	0.01110135	-0.233419183	2.370272209	148.0917583
Arka Government Bond	0.000840429	0.166877174	2.887453935	214.3878481
Arka Money Market Fund	0.000622496	0.251881466	7.008969078	1253.0029
Arka Stable Growth	0.004298434	0.012697679	3.124278019	247.7046109
Arka Eastern European Equity	0.012792742	-0.248315676	2.005746304	108.3426463
POLONIA	0.005885574	0.401138408	-0.192424526	17.2721354

Statistics of funds returns; 11.08.2011 – 25.03.2014 – phase of further growth on certain part of markets after a short-term crash

Society/Investment fund	Standard deviation	Skewness	Excess Kurtosis	JB normality test
PZU Polish Equity	0.010947	-0.86883	5.004058	762.2979
PZU Government Bond	0.001949	-0.16762	2.109217	123.912
PZU Money Market Fund	0.000424	-0.83381	6.733083	1307.135
PZU Stable Growth	0.004626	-0.92413	4.486173	639.5516
PZU Eastern European Equity	0.010823	-0.29901	3.971871	438.2901
Amplico Polish Equity	0.011055	-0.65457	3.75942	430.5118
Amplico Government Bond	0.001508	-1.06926	7.281749	1564.722
Amplico Money Market Fund	0.000785	-4.30183	67.88796	127216
Amplico Stable Growth	0.004025	-1.0412	5.481944	934.2109
Amplico Eastern European Equity	0.006988	-0.14136	2.80618	216.0994
Arka Polish Equity	0.010073	-0.5596	3.626509	391.3138
Arka Government Bond	0.001861	-1.43673	13.79408	5393.489
Arka Money Market Fund	0.000399	-0.62609	19.76005	10650.09
Arka Stable Growth	0.003897	-0.75282	4.175787	535.2957
Arka Eastern European Equity	0.01125	-0.39078	2.572323	196.3517
POLONIA	0.00943	-0.35073	-1.42805	68.76918

APPENDIX 2

Best APARCH, GARCH, IGARCH, EGRACH or GJR type models used in calculation of CVaR; 2.01.2006 – 28.12.2007 – bull market

Society/Investment fund	Conditional mean	Conditional variance	Distribution of random variable
PZU Polish Equity	AR(0)	APARCH(1,1)	Normal
PZU Government Bond	ARMA(1,1)		
PZU Money Market Fund	MA(1)		
PZU Stable Growth	AR(0)	GARCH(1,1)	Normal
PZU Eastern European Equity	AR(0)	APARCH(1,1)	Normal
Amplico Polish Equity	AR(0)	APARCH(1,1)	Normal
Amplico Government Bond	ARMA(3,3)		
Amplico Money Market Fund	MA(2)	GARCH(1,1)	Normal
Amplico Stable Growth	AR(0)	GJR(1,1)	Normal
Amplico Eastern European Equity	AR(0)	APARCH(1,1)	Normal
Arka Polish Equity	AR(0)	GARCH(1,1)	Normal
Arka Government Bond	MA(2)		
Arka Money Market Fund	AR(2)	GARCH(1,1)	*t*-Student
Arka Stable Growth	AR(0)	GARCH(1,1)	Normal
Arka Eastern European Equity	AR(0)	GARCH(1,1)	Normal

Best APARCH, GARCH, IGARCH, EGRACH or GJR type models used in calculation of CVaR; 2.01.2008 – 27.02.2009 – bear market

Society/Investment fund	Conditional mean	Conditional variance	Distribution of random variable
PZU Polish Equity	AR(0)	IGARCH(1,1)	Normal
PZU Government Bond	MA(3)		
PZU Money Market Fund	ARMA(6,7)		
PZU Stable Growth	AR(0)	GARCH(1,1)	Normal
PZU Eastern European Equity	AR(0)	IGARCH(1,1)	Normal
Amplico Polish Equity	AR(1)	IGARCH(1,1)	Normal
Amplico Government Bond	AR(0)	GARCH(,1)	Normal
Amplico Money Market Fund	AR(0)	IGARCH(1,1)	*t*-Student
Amplico Stable Growth	MA(1)	IGARCH(1,1)	Normal
Amplico Eastern European Equity	AR(0)	IGARCH(1,1)	Normal
Arka Polish Equity	AR(0)	IGARCH(1,1)	Normal
Arka Government Bond	ARMA(1,1)	GJR(1,1)	Normal
Arka Money Market Fund	ARMA(1,1)	IGARCH(6,6)	Normal
Arka Stable Growth	ARMA(1,1)	GARCH(1,1)	Normal
Arka Eastern European Equity	AR(0)	GARCH(1,1)	*t*-Student

Best APARCH, GARCH, IGARCH, EGRACH or GJR type models used in calculation of CVaR; 2.03.2009 – 10.08.2011 – growth after decline

Society/Investment fund	Conditional mean	Conditional variance	Distribution of random variable
PZU Polish Equity	AR(1)	APARCH(1,1)	Normal
PZU Government Bond	AR(2)	GJR(1,1)	*t*-Student
PZU Money Market Fund	MA(2)	IGARCH(1,1)	*t*-Student
PZU Stable Growth	AR(1)	GARCH(1,1)	Normal
PZU Eastern European Equity	AR(0)	GARCH(1,1)	Normal
Amplico Polish Equity	AR(1)	GARCH(1,1)	*t*-Student
Amplico Government Bond	AR(0)		
Amplico Money Market Fund	AR(1)	IGARCH(1,1)	*t*-Student
Amplico Stable Growth	MA(1)	GARCH(1,1)	Normal
Amplico Eastern European Equity	MA(1)	GARCH(1,1)	Normal
Arka Polish Equity	ARMA(1,1)	APARCH(1,1)	Normal
Arka Government Bond	AR(1)	GARCH(1,1)	*t*-Student
Arka Money Market Fund	AR(1)	IGARCH(1,1)	*t*-Student
Arka Stable Growth	AR(1)	IGARCH(1,1)	*t*-Student
Arka Eastern European Equity	AR(1)	APARCH(1,1)	Normal

Best APARCH, GARCH, IGARCH, EGRACH or GJR type models used in calculation of CVaR; 11.08.2011 – 25.03.2014 – phase of further growth on certain part of markets after a short-term crash

Society/Investment fund	Conditional mean	Conditional variance	Distribution of random variable
PZU Polish Equity	ARMA(1,2)	GARCH(1,1)	*t*-Student
PZU Government Bond	AR(1)	IGARCH(1,2)	*t*-Student
PZU Money Market Fund	MA(1)	IGARCH(1,1)	*t*-Student
PZU Stable Growth	AR(0)	GARCH(1,1)	Normal
PZU Eastern European Equity	AR(0)	GARCH(1,1)	Normal
Amplico Polish Equity	AR(0)	GARCH(1,1)	Normal
Amplico Government Bond	AR(1)	IGARCH(1,1)	*t*-Student
Amplico Money Market Fund	AR(0)	IGARCH(1,1)	*t*-Student
Amplico Stable Growth	AR(0)	GARCH(1,1)	*t*-Student
Amplico Eastern European Equity	AR(0)	GARCH(1,1)	Normal
Arka Polish Equity	AR(0)	GARCH(1,1)	*t*-Student
Arka Government Bond	MA(1)	GARCH(1,1)	Normal
Arka Money Market Fund	AR(2)	IGARCH(1,1)	*t*-Student
Arka Stable Growth	AR(0)	GARCH(1,1)	Normal
Arka Eastern European Equity	AR(0)	GARCH(1,1)	*t*-Student