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FUNDAMENTAL PORTFOLIO CONSTRUCTION BASED ON SEMI-VARIANCE

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Key words: Markowitz model, fundamental portfolio, semi-variance, Mahalanobis distance.

Abstract

In models for creating a fundamental portfolio, based on the classical Markowitz model, the variance is usually used as a risk measure. However, equal treatment of negative and positive deviations from the expected rate of return is a slight shortcoming of variance as the risk measure. Markowitz defined semi-variance to measure the negative deviations only. However, finding the fundamental portfolio with minimum semi-variance is not possible with the existing methods.The aim of the article is to propose and verify a method which allows to find a fundamental portfolio with the minimum semi-variance. A synthetic indicator is constructed for each company, describing its economic and financial situation. The method of constructing fundamental portfolios using semi-variance as the risk measure is presented. The differences between the semi-variance fundamental portfolios and variance fundamental portfolios are analysed on example of companies listed on the Warsaw Stock Exchange.

WYKORZYSTANIE SEMIWARIANCJI DO BUDOWY PORTFELA FUNDAMENTALNEGO

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Słowa kluczowe: model Markowitza, portfel fundamentalny, semiwariancja, odległość Mahalanobisa.

Abstrakt

W modelu budowy portfela fundamentalnego bazującym na klasycznym modelu Markowitza za miarę ryzyka najczęściej przyjmuje się wariancję. Jednak pewną wadą wariancji jako miary ryzyka jest jednakowe traktowanie dodatnich i ujemnych odchyleń względem oczekiwanej stopy zwrotu. Do mierzenia tylko odchyleń ujemnych Markowitz zdefiniował semiwariancję. Znalezienie portfela fundamentalnego o minimalnej semiwariancji nie jest możliwe z wykorzystaniem istniejących metod. Celem artykułu jest zaproponowanie i zweryfikowanie metody pozwalającej na znalezienie portfela fundamentalnego o minimalnej semiwariancji. Dla każdego analizowanego przedsiębiorstwa wyznaczono wskaźnik syntetyczny, opisujący jego sytuację ekonomiczno-finansową. Zaproponowano metodę budowy portfela fundamentalnego o minimalnej semiwariancji. Przeanalizowano różnice między portfelami fundamentalnymi o minimalnej semiwariancji a portfelami o minimalnej wariancji na przykładzie spółek notowanych na Giełdzie Papierów Wartościowych w Warszawie.

Introduction

Risk in development models of a fundamental portfolio is measured by rate of return variance (TARCZYŃSKI 1995, p. 99, TARCZYŃSKI 2002, p. 115), (RUTKOWSKA 2011, p. 554). One of the drawbacks of variance as a measure of risk is that negative and positive deviations from the expected rate of return are treated in the same manner. In fact, negative deviations are undesirable, while positive ones create an opportunity for a higher profit.

There has been a notion in the literature that variance can be used to measure risk when rate of return distribution is normal, or at least symmetrical, (GALAGEDERA, BROOKS 2007, p. 215–216) or investor's utility functions are square functions (ELTON, GRUBER 1998, p. 263). The square utility function has some undesirable properties, whereby it does not describe investors' behaviours correctly. First of all, it reaches the maximum value for a certain rate of return and it decreases with an increasing rate of return, which is in open contradiction to investors' preferences, who always want to have more, rather than less. In contrast to the variance the utility function for semi-variance is increasing function for all rates of return (MARKOWITZ 1991, p. 290).

Studies conducted on capital markets have shown that the rate of return distributions for some quoted companies are not normal, or even symmetrical distributions, exempli gratia power-low or log-normal (MANDELBROT 1997, ADCOCK, SHUTES 2005, p. 402–414, POST, VIET 2006, p. 824). When the semi-variance is applied, no assumption about rate of return distribution is required (HARLOW, RAO 1989). Furthermore, it can be shown that when the rate of return distribution is normal, semi-variance is a better measure of risk than variance (RUTKOWSKA-ZIARKO 2007, p. 105-116).

The theoretical objective of the article is to propose a method which allows to find a fundamental portfolio with the minimum semi-variance, which is not possible with the existing methods. In terms of application, the objective of the article is to verify the method.

Taxonomic measure of attractiveness of investments

When the portfolio is constructed based on fundamental analysis, it is necessary to quantitatively determine the economic and financial situation of the company. To this end, financial ratios are used, calculated from the financial reports published by companies. The problem is to present the economic and financial situation of a company with just one index. To this end, a synthetic development measure may be used, based on selected financial ratios. A synthetic development measure was first used for portfolio construction by Tarczyński (TARCZYŃSKI 1995, p. 97–99). He referred to the measure as the taxonomic measure of attractiveness of investments – TMAI (TARCZYŃSKI 1995, p. 97). In fundamental portfolio construction models, companies are usually arranged according to the Euclidean distance (TARCZYŃSKI 1995, p. 97, TARCZYŃSKI 2002, p. 98, RUTKOWSKA 2011, p. 555–556). Due to a possible correlation among diagnostic variables, the Mahalanobis distance (BALICKI 2009, p. 216) is a more appropriate distance measure between companies, compared with Euclidean distance (RUTKOWSKA-ZIARKO 2013). Other synthetic measures have been used in capital market studies, such as: *generalized distance measure* (GDM) and *absolute development level ratios* (BZW) (ŁUNIEWSKA 2005 p. 469). The advantage of the GDM measure lies in that it can be used for variables measured in weaker scales (WALESIAK, DUDEK 2010, p. 186). The Mahalanobis distance has been used in this article to determine TMAI.

Four financial ratios have been taken as diagnostic variables in this paper. Three of them described the financial situation of the companies under study: current ratio (CR), return of assets ratio (ROA) and debt ratio (DR). The study also took into account the market price-earning ratio (P/E). Studies of capital markets have revealed a negative correlation between the value and future share price increases (BASU 1977, p. 663). Therefore, P/E was regarded as a destimulant and replaced with E/P:

$$E/P = \frac{1}{P/E} \tag{1}$$

CR and ROA ratios were regarded as stimulants, whereas DR – as a destimulant. DR was replaced with a corresponding stimulant (DR').

$$\mathrm{DR'} = \frac{1}{\mathrm{DR}} \tag{2}$$

Subsequently, a reference standard was created with which the analysed companies were compared. When Mahalanobis distances are used, it is not necessary to standardise diagnostic variables, as is the case with Euclidean distances (BALICKI 2009, p. 216).

Let w_{il} denote values of diagnostic variables, where $l = 1,..., m$ is the number of diagnostic variables considered. For each diagnostic variable, the highest observed value of (w_{0l}) is sought (HELLWIG 1968, p. 323–326):

$$w_{0l} = \max_i \{w_{il}\} \tag{3}$$

An abstract point $P_0(w_{0l})$ was taken as the reference standard; its coordinates assume the highest values of the diagnostic variables after transformation of variables into stimulants. The distance for each company, that is candidate for portfolio, relative to the idealised target $P_0(w_{0l})$ was calculated.

The TheMahalanobis distance could be calculated as follows (MAHALANOBIS 1936, p. 50):

$$MQ = \sqrt{(W_i - W_l) \cdot C^{-1}(W_i - W_l)^T} \tag{4}$$

where W_i is a row vector, $W_i = [w_{i1},..., w_{i4}]$, W_l is a row vector, representing "the ideal quoted company" $W_i = [w_{01},..., w_{04}]$, C is covariance matrix for diagnostic variables.

Mahalanobis distance was used to determine the taxonomic measure of attractiveness of investments for each company (TARCZYŃSKI 2002, p. 98):

$$\mathrm{TMAI_i} = 1 - \frac{Q_i}{\max_i \{Q_i\}} \tag{5}$$

Fundamental portfolio and semi-variance

By combining elements of the portfolio and fundamental analysis, an additional condition can be introduced to the classic Markowitz model which ensures that the portfolio will contain only companies with good economic and financial standing. The fundamental portfolio is an efficient portfolio if, for a specific average rate of return and average TMAI, the variance calculated for it is the lowest.

The model of construction of a fundamental portfolio, which has been used in the study, is a modification of the classic Markowitz model (MARKOWITZ 1952, p. 81). A limiting condition has been introduced to the portfolio construction model, according to which the TMAI total, weighted by contribution of shares of a specific company in the portfolio, must achieve at least the level set by the investor. The construction model of a fundamental portfolio will have the following form:

minimise the portfolio variance (RUTKOWSKA-ZIARKO 2011, p. 554):

$$S_p^2 = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j \operatorname{cov}_{ij} \tag{6}$$

with the limitations:

$$\sum_{i=1}^{k} x_i = 1 \tag{7}$$

$$\sum_{i=1}^{k} x_i \bar{z}_i \geq \gamma \tag{8}$$

$$x_i \geq 0 \; i = 1,\dots, k \tag{9}$$

$$\text{TMAI}_p = \sum \text{TMAI}_i x_i \geq \text{TMAI}_\gamma \tag{10}$$

where: S_p^2 is variance of rate of return; cov_{ij} is covariance between security i and security j γ-target rate of return, assuming that $\gamma \leq \max \bar{z}_i$; $\bar{z}_i$ – mean rate of return on security i; x_i – contribution by value of the i-th share in the portfolio; $TMAI_\gamma$ – the sum of TMAI, required by the investor, weighted by the contribution of shares in the portfolio.

Considering the drawbacks of variance as a measure of risk, a monograph on the choice of a portfolio by MARKOWITZ (1959, p. 188–189) suggests semi-variance of the assumed rate of return $dS^2(\gamma)$ as a measure of risk which is an alternative to variance:

$$dS^2(\gamma) = \frac{\sum_{t=1}^{m} d_t^2(\gamma)}{m-1}, \; t = (1,2,\dots, m) \tag{11}$$

where:

$$d_t(\gamma) = \begin{cases} 0 & \text{for} \quad z_t \geq \gamma \\ z_t - \gamma & \text{for} \quad z_t < \gamma \end{cases} \tag{12}$$

When semi-variance of an investment portfolio is determined, semi-covariances of the rates of return of shares which it comprises are used:

$$dS_p^2\ (\gamma) = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j\ d_{ij}(\gamma) \tag{13}$$

where: $dS_p^2\ (\gamma)$ – semi-variance of the portfolio rates of return; $d_{ij}(\gamma)$ semi-covariance of the rate of return for the i-th and the j-th share.

When semi-covariance is determined, it is noted in which periods the rate of return is higher and in which it is lower than the level assumed by the investor.

$$d_{ij}\ (\gamma) = \frac{1}{m-1} \sum_{t=1}^{m} d_{ijt}\ (\gamma) \tag{14}$$

where:

$$d_{ijt}(\gamma) = \begin{cases} 0 & \text{for } z_{pt} \geq \gamma \\ (z_{it} - \gamma)(z_{jt} - \gamma) & \text{for } z_{pt} < \gamma \end{cases} \tag{15}$$

where:

$$z_{pt} = \sum_{i=1}^{m} x_i z_{it},\ t = (1,2,...,\ m) \tag{16}$$

Determination of effective portfolios for the risk understood to denote a possibility of achieving a lower rate of return than the assumed value is reduced to minimising semivariance of the assumed rate of return at the predetermined value of γ, therefore, to solving the following optimising problem:

minimise the semi-variance of portfolio rate of return:

$$dS_p^2\ (\gamma) = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j\ d_{ij}(\gamma)$$

with the limitations (7–10).

When seeking an effective portfolio for the risk measured with semivariance, the sum of squares of "downward" deviations from the assumed rate of return is minimised, while there are no limitations imposed on "upward" deviations. Further in the article, the fundamental portfolio with the minimal variance will be referred to as VFP, while that with the minimum semi-variance – as SFP.

Using semi-variance to determine effective portfolios creates considerable problems because when semi-covariance of rates of return are determined

$d_{ij}(\gamma)$ one has to know in which periods the rate of return of the entire portfolio was lower than the assumed value, and this depends both on the assumed rate of return and on the portfolio composition. This makes determination of effective portfolios for semi-variance of the assumed rate of return more complicated than for variance. In order to determine the composition of Markowitz's portfolio for any γ, it is enough to know co-variance cov_{ij} and mean rates of return $\bar{z}_i$, those parameters are estimated on a one-off basis and they do not depend either on the portfolio composition or on the assumed rate of returnγ. On the other hand, when a portfolio with the minimum semi-variance is determined, each time the composition of the portfolio or the assumed rate of return γ changes, semi-covariance of rates of return $d_{ij}(\gamma)$ should be re-estimated. In order to determine an effective fundamental portfolio, which minimises semi-variance of the assumed rate of return, modification of the iterative algorithm (used to build a portfolio with the minimum semi-variance) was applied (RUTKOWSKA-ZIARKO 2005, p. 72–77). Starting with the FTP portfolio, the following procedure is reiterated until self-consistency[1] of the portfolio composition has been achieved:

1. Determination of the rates of return of portfolio z_{pt} within time units according to (16).
2. Determination of semi-covariances of rates of return $d_{ij}(\gamma)$ (14–15).
3. For the semi-covariance of rates of return $d_{ij}(\gamma)$ determined in point 2. – minimise the semi-variance of portfolio rate of return:

$$dS_p^2\,(\gamma) = \sum_{i=1}^{k}\sum_{j=1}^{k} x_i x_j\, d_{ij}(\gamma)$$

with the limitations (7–10).

Further in the article, this procedure will be referred to as the MFP procedure.

It should be emphasised that determination of the composition of the VFP portfolio and the semi-variance minimising portfolio in subsequent passages through point 3 is an issue which is independent of the proposed procedure. In this study, a ready-to-use optimising package, named WinQSB, has been used. It is used as a sub-program and it could be replaced with any algorithm of non-linear programming, which would not affect the SFP procedure.

[1] Self-consistency is understood as stabilisation of the portfolio composition at a set level of precision.

Empirical results

The study covered 10 of the largest and most liquid companies listed on The Warsaw Stock Exchange (included in the WIG20 index), excluding financial institutions[2]. The study was based on quarterly rates of return calculated based on daily closing prices during the period from January 1, 2010 until March 22, 2011. Rates of return were computed as relative increases in prices of stocks according to the formula:

$$R_{it} = \frac{N_{i,t+s} - N_{it}}{N_{it}} \cdot 100\% \qquad (17)$$

where R_{it} is the rate of return on security i at time t, s is the length of the investment process expressed in days, N_{it} is the listed value of security i at time t, $N_{i,t+s}$ is the listed value of security i after s days of investing started at time t.

The share closing price on March 22, 2011, was taken as the market share price of a company. Financial ratios were calculated for each company based on annual financial reports for 2010.

For each of the analysed companies taxonomic measure of attractiveness of investments was determined. Based on time series of rates of return mean rate of return, variance and semi-variance were calculated. The profitability, risk and taxonomic measures of attractiveness of investments are presented in Table 1.

Table 1

Profitability, risk and taxonomic measures of attractiveness of investments

Company	Mean (%)	Variance	dS^2 (4)	dS^2 (8)	dS^2 (12)	TMAI	Rank
ACP	-2.25	18.25	56.67	123.65	222.06	0.5766	3
KGH	16.26	299.22	45.20	72.67	110.73	0.7195	1
LTS	9.22	137.19	19.70	45.33	88.46	0.1519	8
LWB	12.41	208.32	16.29	37.15	72.60	0.4287	5
PBG	0.03	36.78	45.71	99.02	180.54	0.6674	2
PGE	0.39	27.81	37.38	85.61	163.16	0.4526	4
PGN	8.37	53.24	6.62	21.80	54.07	0.1949	7
PKN	1.61	86.57	56.60	109.28	186.72	–	10
TPS	-0.15	72.87	73.24	133.62	219.67	0.2819	6
TVN	3.11	132.35	51.15	101.90	176.96	0.1166	9

Source: the author's own calculation.

[2] The three-letter abbreviations used at the Warsaw Stock Exchange are used in the paper instead of the full names of stock issuers.

During the analysed period, the highest TMAI values was calculated for the KGH company; at the same time, its risk, measured as the variance of rates of return was the highest. Considering the semi-variance for different levels of γ, risk was at a medium level as compared to the other analysed companies. The variance for the ACP company was the lowest; however, the values of semi-variance were very high.

Efficient fundamental portfolios with the minimum variance values (VFP) were built as well as fundamental portfolios with the minimum semi-variance (SFP) for selected levels of target rate of return ($\gamma = 4, 8, 12\%$) and $TMAI_\gamma = 0.5$. For all the portfolios, the limitation concerning the required level of TMAI was an active limitation. The composition of the determined portfolios (by value) and their selected characteristics are presented in Table 2, 3 and 4.

Table 2

Efficient fundamental portfolio for $\gamma = 4\%$ and $TMAI_\gamma = 0.5$

Issuer	Starting portfolio	Portfolio composition in the *i*-th iteration						
		1	2	3	4	5	6	7
ACP	0.352	0.428	0.386	–	–	–	–	–
KGH	0.122	0.153	0.159	–	–	–	–	–
LTS	0.067	–	–	–	0.050	0.059	0.064	0.064
LWB	0.169	0.230	0.284	0.680	0.718	0.722	0.722	0.722
PGE	0.132	–	–	–	–	–	–	–
PGN	–	–	–	–	–	–	–	–
PKN	–	0.102	0.057	–	0.014	0.008	0.003	0.003
TPS	–	–	–	–	–	–	–	–
PBG	0.122	0.087	0.103	0.140	0.002	–	–	–
TVN	0.036	–	0.011	0.180	0.216	0.211	0.211	0.211
	starting portfolio	profitability, risk and TMAI in the *i*-th iteration						
		1	2	3	4	5	6	7
Average rate of return	4.000	5.216	5.742	8.987	10.139	10.229	10.233	10.233
Variance	20.362	31.114	36.458	85.369	97.513	97.967	97.770	97.770
Semi-variance of 4%	11.698	10.470	10.333	6.068	5.341	5.321	5.324	5.324
$TMAI_p$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Source: The author's own calculation with WinQSB.

The SFP portfolios were determined by iteration by following the SFP procedure, assuming that self-consistency of the portfolio is achieved after its composition is stabilised at the accuracy of 3 decimal digits. The fundamental portfolio with the minimum variance (VFP) was each time taken as the starting portfolio. The final solution was achieved in the seventh iteration, because – considering the assumed accuracy – the iteration does not contain changes in the portfolio composition or the value of semi-variance at the assumed rate of return of 4%. Shares for the portfolio in each iteration were chosen from among the initial set of 10 quoted companies. The largest decrease in semi-variance took place in the first and third iteration; it was then that the largest changes concerning the companies making up the portfolio and the proportions of their contributions took place. However, no change took place in the companies in the portfolio starting from the fifth iteration; only proportions of shares changed slightly and the decrease in semi-variance was small.

Table 3

Efficient fundamental portfolio for γ = 8% and $TMAI_\gamma$ = 0.5

Issuer	Starting portfolio	Portfolio composition in the *i*-th iteration				
		1	2	3	4	5
ACP	0.147	–	–	–	–	–
KGH	0.296	–	0.016	0.010	0.010	0.010
LTS	0.061	–	0.021	0.058	0.059	0.059
LWB	0.189	0.664	0.703	0.706	0.707	0.707
PGE	0.083	–	–	–	–	–
PGN	–	–	–	–	–	–
PKN	0.075	–	0.039	–	–	–
TPS	–	–	–	–	–	–
PBG	0.149	0.063	–	–	–	–
TVN	–	0.273	0.221	0.227	0.225	0.225
	starting portfolio	profitability, risk and TMAI in the *i*-th iteration				
		1	2	3	4	5
Average rate of return	8.000	9.084	10.193	10.152	10.168	10.168
Variance	56.352	82.746	97.640	95.190	95.478	95.478
Semi-variance of 8%	31.543	22.670	20.178	20.109	20.102	20.102
$TMAI_p$	0.5	0.5	0.5	0.5	0.5	0.5

Source: The author's own calculation with WinQSB.

In the case of γ = 0.8% final solution was achieved in the fifth iteration. The largest decrease in the semi-variance took place in the first iteration.

In the case of γ = 12% final solution was achieved in the fourth iteration. The largest decrease in the semi-variance took place in the first iteration.

SFP portfolios are safer than the initial portfolio (lower semi-variance than the assumed rate of return). It means in practice that on average, deviations below the target rate of return for SFP portfolios are smaller in comparison to their VFP counterparts. Moreover, SFP portfolios for γ = 4, 8% have a higher

Table 4

Efficient fundamental portfolio for γ = 12% and $TMAI_\gamma$ = 0.5

Issuer	Starting portfolio	Portfolio composition in the *i*-th iteration			
		1	2	3	4
ACP	–	–	–	–	–
KGH	0.469	0.129	0.173	0.174	0.174
LTS	0.046	–	–	–	–
LWB	0.218	0.700	0.650	0.649	0.649
PGE	–	–	–	–	–
PGN	–	–	–	–	–
PKN	0.150	0.130	0.108	0.108	0.108
TPS	–	–	–	–	–
PBG	0.116	–	–	–	–
TVN	–	0.041	0.069	0.069	0.069
	starting portfolio	profitability, risk and TMAI in the *i*-th iteration			
		1	2	3	4
Average rate of return	12	12	12	12	12
Variance	115.527	135.824	130.489	130.394	130.394
Semi-variance of 12%	62.716	53.232	52.776	52.774	52.774
$TMAI_p$	0.5	0.5	0.5	0.5	0.5

Source: The author's own calculation with WinQSB.

average rate of return. SFP portfolios have different compositions than VFP portfolios. SFP portfolios differ from VFP portfolios in terms of the shares present in them and the proportions between their contributions. For each assumed rate of return, part of the companies present in both types of portfolios are the same.

Conclusion

The results have shown that adopting a specific measure of risk significantly affects which achievable portfolios will be regarded as effective. The differences between effective fundamental portfolios with the minimum variance and those minimising semi-variance of the assumed rate of return are particularly distinct for low assumed rates of return. The determined SFP portfolios have lower semi-variance of the assumed rate of return than VFP portfolios, i.e. they are safer. Moreover, those are more profitable portfolios for lower assumed rates of return.

The advantage of the method is the possibility of using one of the available applications of non-linear or square programming to determine portfolios with the minimum semi-variance.

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References

ADCOCK C.J., SHUTES K. 2005. *An analysis of skewness and skewness persistence in three emerging markets*. Emerging Markets Review, 6: 396–418.

BALICKI A. 2009. *Statystyczna analiza wielowymiarowa i jej zastosowania społeczno-ekonomiczne*. Wydawnictwo Uniwersytetu Gdańskiego, Gdańsk.

BASU S. 1977. *Investment performance of common stocks In relation to their price-earnings ratios: a test of the efficient market hypothesis*. Journal of Finance, 3: 663–682

GALAGEDERA U.A., BROOKS R.D. 2007. *Is co-skewness a better measure of risk in the downside than downside beta? Evidence in emerging market data.* Journal of Multinational Financial Management, 17: 214–230.

HARLOW W.V., RAO R.K.S. 1989. *Asset pricing in a generalized mean-lower partial moment framework: theory and evidence*. Journal of Financial and Quantitative Analysis, 24: 285–311.

HELLWIG Z. 1968. *Zastosowanie metody taksonomicznej do typologicznego podziału krajów ze względu na poziom ich rozwoju oraz zasoby i strukturę wykwalifikowanych kadr*. Przegląd Statystyczny, 4: 323–326.

ELTON E.J., GRUBER M.J. 1998. *Nowoczesna teoria portfelowa i analiza papierów wartościowych*. WIG PRESS.

ŁUNIEWSKA M. 2005. *Evaluation of Selected Methods of Classification for the Warsaw Stock Exchange*. International Advances in Economic Research, 11: 469–481.

MANDELBROT B., FISHER A., CALVET L. 1997. *A Multifractal Model of Asset Returns*. Working Papers-Yale School of Management's Economics Research Network, 1997: 1–33.

MAHALANOBIS P.C. 1936. *On the Generalized Distance in Statistics*. Proceedings of the National Institute of Science. India, 12: 49–55.

MARKOWITZ H. 1952. *Portfolio selection*. Journal of Finance, 7: 77–91.

MARKOWITZ H. 1959. *Portfolio selection: efficient diversification of investments*. John Wiley and Sons, New York.

MARKOWITZ H. 1991. *Portfolio selection: efficient diversification of investments*. Blackwell, Malden, Massachusetts.

POST T., VAN VLIET P. 2006. *Downside risk and asset pricing*. Journal of Banking and Finance, 30: 823–849.

RUTKOWSKA-ZIARKO A, 2005. *Metody znajdowania portfela efektywnego dla semiwariancji.* Badania operacyjne i decyzyjne, 3–4: 63–83.

RUTKOWSKA-ZIARKO A. 2007. *Wykorzystanie wariancji i semiwariancji do budowy portfela akcji przy normalności rozkładów stóp zwrotu*. Przegląd Statystyczny, 4: 105–116.

RUTKOWSKA-ZIARKO A. 2011. *Alternatywna metoda budowy fundamentalnego portfela papierów wartościowych.* Taksonomia 18 – Klasyfikacja i analiza danych – teoria i zastosowania: p. 551–559.

RUTKOWSKA-ZIARKO A. 2013. *Fundamental portfolio construction based on Mahalanobis distan*ce. Studies in Classification. Data Analysis and Knowledge Organization: in press.

TARCZYŃSKI W. 2002. *Fundamentalny portfel papierów wartościowych*. PWE, Warszawa.

TARCZYŃSKI W. 1995. *O pewnym sposobie wyznaczania składu portfela papierów wartościowych*. Przegląd Statystyczny, 1: 91–106.

WALESIAK M., DUDEK A. 2010. *Finding Groups in Ordinal Data: An Examination of Some Clustering Procedures*. Studies in Classification. Data Analysis. and Knowledge Organization. Part 2: p. 185–192.