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TIME SERIES DECOMPOSITION AS A METHOD OF MEASURING CAPITAL MARKETS CONVERGENCE

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Abstract

The aim of the article is to present time series decomposition as a method of measuring capital markets convergence. As an example, convergence of two different sets of markets are measured using this methodology. On the basis of this research, it has been established that time series decomposition of the market indices can prove or reject a hypothesis of moving indices in similar directions over a period of time.

DEKOMPOZYCJA SZEREGÓW CZASOWYCH JAKO METODA POMIARU ZBIEŻNOŚCI RYNKÓW KAPITAŁOWYCH

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Kody JEL: C32, E32, G15.

Słowa kluczowe: analiza szeregów czasowych, międzynarodowe rynki kapitałowe, zbieżność rynków kapitałowych

Abstrakt

Celem artykułu jest prezentacja dekompozycji szeregów czasowych jako metody pomiaru zbieżności rynków kapitałowych. Jako przykład zbieżność dwóch par rynków kapitałowych zmierzono z zastosowaniem tej metodologii. Na bazie tych badań stwierdzono, że dekompozycja szeregów

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czasowych składających się z wartości indeksów cenowych rynków kapitałowych może służyć do potwierdzenia lub odrzucenia hipotez dotyczących poruszania się indeksów w podobnych kierunkach w badanych okresach czasu.

Introduction

In the capital markets there is said to be "convergence" between markets when investors regard the instruments traded in these separate markets as substitutes. Prices of these instruments will in turn show a tendency of moving in the same direction. As examples, we can show cases in which an increase in stock prices follows an increase in bond prices. Or, examples where stock prices on different international markets are moving in the same direction. According to another definition of "convergence", it occurs when capital can move between markets which have no transactional barriers between them. In such instances, the price of risk (the reward which investors receive for taking this risk) on different markets is equal for the same kind of assets. The opposite term is "divergence" which describes the situation when the prices of similar instruments are moving in different directions. The objective of the study described in this article is to demonstrate how convergence can be measured using one of the quantitative methods – time series decomposition. The analysis of convergence is based on comparison of the value of the divergence factor calculated as a sum of the differences between extracted values of irregular component, for two time series.

Analysis of literature on time series decomposition

Time series decomposition is used by economics and finance researchers to examine different phenomena that can be described by time series. Examples presented by authors (Cooraj, 2008; Enders, 2003) include such time series as sales and/or profits of companies, production facilities outputs, various indices (such as retail or consumer prices indices). The application of the method to measuring stock markets convergence is a novelty and has not been noticed in the existing literature.

Research methodology

A time series is a series of figures or values recorded over time. There are several components of a time series which it may be necessary to identify:
– a trend;
– seasonal variations or fluctuations;
– cyclical variations;

– non-recurring, random variations. These may be caused by unforeseen circumstances, such as political events (e.g. elections, change in the government etc.), a war or technological changes.

The trend is the underlying long-term movement over time in the values of the data recorded. Basically, there are three types of trend: a downward trend, an upward trend and a static trend, where there is no clear movement up or down along the time series.

Seasonal variations are short term fluctuations in recorded values, due to different circumstances which affect results at different times of year (or of a few-years periods), on different days of the week or at different times of day. "Seasonal" is a term which may appear to refer to the seasons of the year but its meaning in time series analysis is somewhat broader (the seasons in question don't have to cover the actual seasons of the year and may be shorter or longer, however it is safe to assume that in case of most of stock of exchange indices movements seasons correspond to certain seasons of the year – e.g. stock indices usually go up in May and December etc.)

Cyclical variations usually refer to the variations of the frequency higher than 1 year. As an example, we can bring the data on employment using quarterly data. For stock exchanges indices such variations are hard to identify and are not meaningful within the long timespan selected for research. Therefore, from now on, we will exclude any reference to this component.

In practice, a time series could incorporate all three components. For efficient decomposition, the three components have to be isolated. We can begin the process of isolating each feature by summarizing the components of a time series by equation 1 (Hamilton, 1994; Valkanov, 2003):

$$Y = T + S + I \tag{1}$$

(we have excluded the cyclical component)

where:

Y – the actual time series,
T – the trend series,
S – the seasonal component,
I – the irregular (random) component.

The alternative method is to use the multiplicative model whereby each actual figure is expressed as a proportion of the trend. Sometimes this method is called the proportional model. This model summarises a time series as $Y = T \cdot S \cdot I$. Please note that the trend component will be the same whichever model is used but the values of the seasonal and residual components will vary according to the model being applied. For the purpose of our study we will use the additive model described previously.

The first step of the time series decomposition is isolating the trend. There are three principal methods of finding a trend. The trend line can be drawn by eye on a graph in such a way that it appears to lie evenly between the recorded points, that is, a line of best fit drawn by eye. Alternatively, a statistical technique known as linear regression by the least squares' method can be used to calculate a line of best fit. However, the most frequently used method is a technique known as moving averages. This method attempts to remove seasonal variations from actual data by a process of averaging in order to produce trend values. The methods described above are useful for finding a linear trend. There are also different types of trend such as polynomial, logarithmic or exponential, for which different methods of decomposition are used such as exponential smoothing or other, but their usage is outside of the scope of this study.

A simple moving average is an average of the results of a fixed number of periods. Since it is an average of several time periods it is related to the mid-point of the overall period. In order to calculate the simple moving average, we can use the formula 2 (Frątczak, 2015; Frątczak & Korczyński, 2013):

$$X_t = \frac{1}{2q+1} \sum_{r=-q}^{q} x_{t+r,} \quad (t = q + 1, q + 2, \ldots, n - q) \tag{2}$$

where:

X_t – moving average,

x_i – next value from time series,

q – a natural number, representing fixed, odd number of periods, divided by 2 and rounded down,

t – moving average counter,

The formula presented above concerns a case when moving averages are taken of the results in an odd number of time periods, and the average is related to the mid-point of the overall period. If a moving average were taken of the results in an even number of time periods, the basic technique would be the same, but the mid-point of the overall period would not relate to a single period. The trend line average figures need to relate to a particular time period, otherwise, seasonal variations cannot be calculated. To overcome this difficulty, we take a moving average of the moving average (we repeat the process). For measuring capital market convergence periods equal to calendar months have been taken.

Once a trend has been established, by whatever method, we can find the seasonal variations. The additive model for time series analysis is $Y = T + S + I$. We can therefore write $Y - T = S + I$. In other words, if we deduct the trend series from the actual series, we will be left with the seasonal and irregular (residual) components of the time series.

In order to identify the irregular (residual) component, we have to calculate the average values of the sums of S and I components for every month of the cycle (we assume that the full cycle lasts 12 calendar months). Then we have to deduct from the sums the seasonal component alone. In order to calculate the seasonal component alone, first we calculate the sums of the $S + I$ values for every calendar month over the whole period and divide by the number of the cycles in the whole period (years in the period examined) to arrive at the average value for each month. One more step is necessary in case the sum of the calculated seasonal component does not total to zero. We divide the excess by the number of months in the cycle and deduct the result from the sums of S and I components for each month of the cycle to arrive at adjusted seasonal component. After deduction of the values of the adjusted seasonal component from the $S + I$ values, we are left with the irregular (residual) component alone. (Cooraj, 2008; Enders, 2003). Time series decomposition can be used for making forecasts (Peck & Devore, 2012; Jóźwiak & Podgórski, 2000). These can be made as follows:

– trend line is found by calculating the moving averages and the trend line values are plotted on the graph;

– the trend line is extended so that readings for points in time outside of time covered by the original data can be taken. This method is known as extrapolation;

– the readings found using the extrapolated trend line are adjusted by the average seasonal variation applicable to the future period.

Alternatively, time series decomposition can be used for measuring the convergence of two (or more) time series. For such exercise we use the irregular component alone. A good example can be measuring the convergence of capital markets stock price indices. In order to compare the irregular component for different stock exchanges we need to standardize these values first (deduct the arithmetic mean and divide by the standard deviation). As the movements of the irregular component of the stock price indices is a measure of reaction of the stock exchanges to various political or macroeconomic events, we can assume that these are also a measure of the stock markets convergence for the different markets. In order to measure the convergence of capital markets the values of standardized irregular component need to be compared in months where this component is higher than 1 or lower than -1 and the difference in the pairs of the component values for the stock exchanges selected need to be computed. Then the sum of the absolute values of the differences needs to be calculated and compared between selected pairs (or sets in case we compare more than 2 time series). As a result, we can determine that the sets with larger value of the sum of the differences are less convergent than the sets with a smaller value of the sum of the differences, so we can call the sum of the differences a divergence factor. The formula for the divergence factor is as follows 3:

$$D = \sum_{1}^{n} |d_n| \tag{3}$$

where:

D – divergence factor,

n – number of months where the values of standardized irregular component is higher than 1 or lower than -1,

$|d_n|$ – the absolute value of the n-th difference between the 2 markets' irregular components in cases described above.

Description of the research results

To illustrate the usage of this technique, the convergence of 2 capital markets has been measured (in 2 examples). Selected pairs of capital markets were NYSE market and BSE market in India and NYSE market and SSE market

Table 1

Sums of the differences between NYSE/BSE and NYSE/SSE irregular components of the indices and the convergence factors

Month	Diff BSE/ NYSE	Month	Diff BSE/ NYSE	Month	Diff SSE/ NYSE	Month	Diff SSE/ NYSE
08.2007	1.1291	05.2010	0.1537	08.2007	2.3091	03.2010	1.0592
09.2007	1.3802	06.2010	1.772	09.2007	1.6536	05.2010	0.2759
10.2007	0.2823	08.2010	1.5631	10.2007	1.958	06.2010	0.2893
11.2007	2.4587	09.2010	1.2008	12.2007	2.047	08.2010	1.5631
12.2007	3.9615	10.2010	1.4286	03.2008	0.3793	01.2011	1.1154
01.2008	1.1083	12.2010	1.1605	05.2008	1.9195	02.2011	1.4633
02.2008	1.1561	01.2011	1.1154	06.2008	1.2432	04.2011	1.1337
03.2008	1.0363	02.2011	1.4633	07.2008	1.269	08.2011	1.006
05.2008	1.9195	04.2011	1.1337	08.2008	2.3829	09.2011	2.5914
06.2008	2.3351	08.2011	1.006	09.2008	1.3338	02.2012	1.2615
07.2008	1.269	09.2011	1.466	10.2008	0.0623	05.2012	1.6724
08.2008	1.041	11.2011	1.3488	11.2008	0.9775	04.2014	1.0669
09.2008	1.3338	12.2011	1.8556	12.2008	0.338	05.2014	1.1288
10.2008	0.4343	02.2012	0.2315	01.2009	1.7977	04.2015	2.6079
11.2008	0.1931	05.2012	0.276	02.2009	3.1237	05.2015	3.2402
12.2008	0.0709	07.2012	1.1511	03.2009	2.3345	06.2015	2.4254
01.2009	0.1298	01.2013	1.0914	04.2009	1.4855	08.2015	1.0914
02.2009	1.2003	07.2013	1.0034	07.2009	1.8391	09.2015	0.196
03.2009	0.6034	08.2013	1.6257	09.2009	1.2947	01.2016	0.2649
04.2009	0.2497	02.2015	1.293			06.2016	0.0716
05.2009	1.0877	09.2015	1.1619				**55.273**
09.2009	1.2947	01.2016	1.0095				
03.2010	1.0592	02.2016	0.2029				
			52.448				

Source: own computation on the basis of data from WFE website.

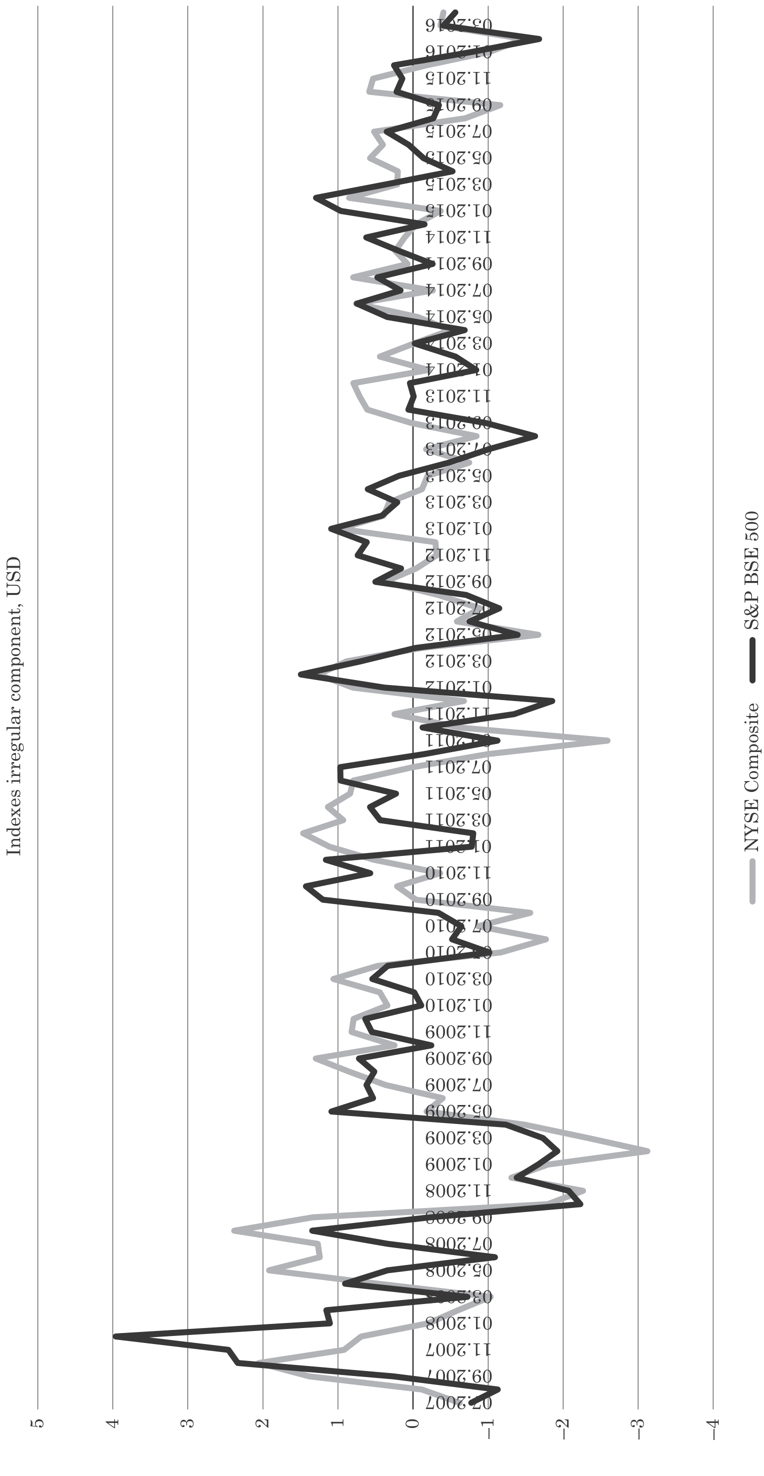

Fig. 1. Irregular components of NYSE market and BSE market indices

Source: own computation on the basis of data from WFE website.

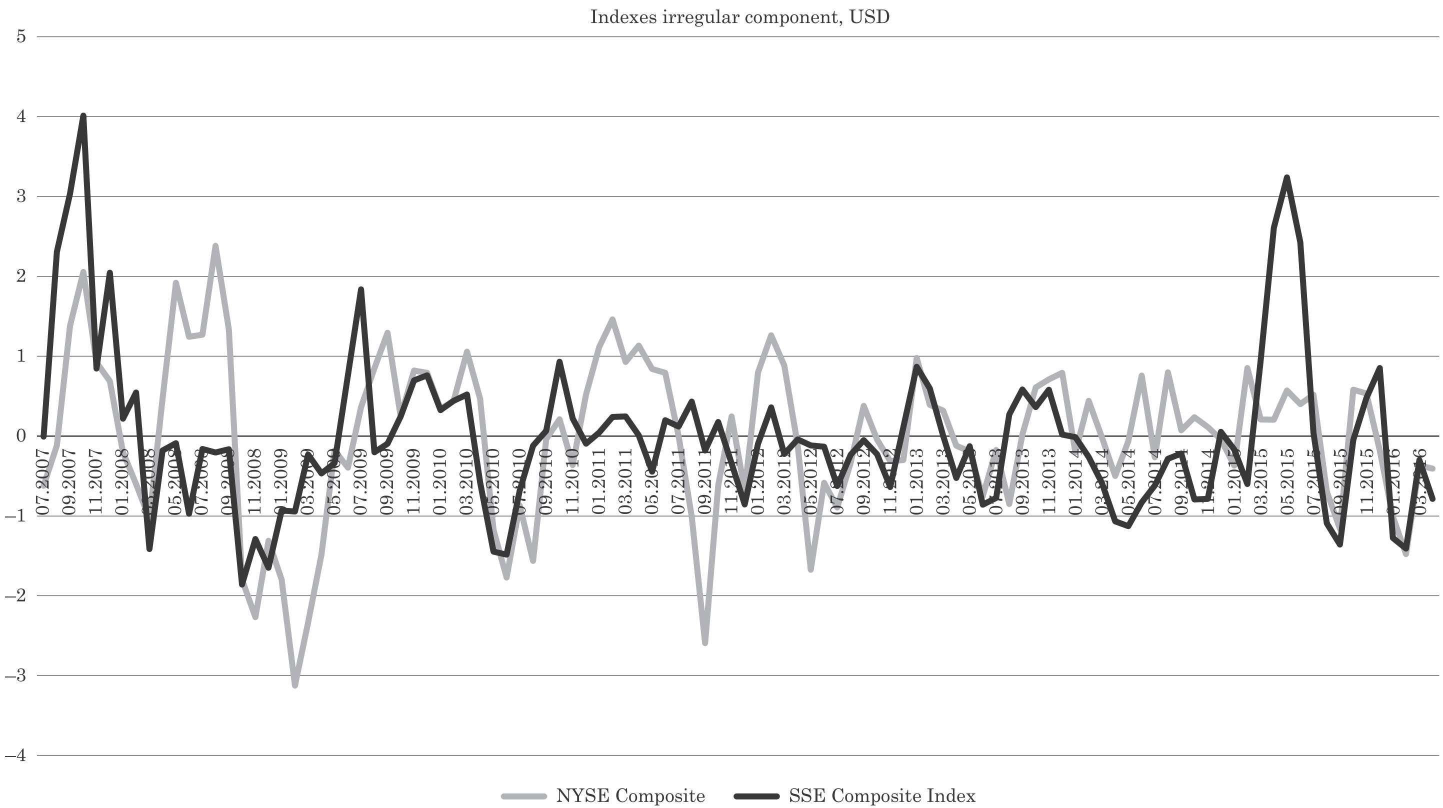

Fig. 2. Irregular components of NYSE market and SSE market indices

Source: Own computation on the basis of data from WFE website.

in China and a period between July 2007 and April 2016 has been chosen. The indices selected were NYSE Composite (for the NYSE market), S&P BSE 500 (for the Indian market) and SSE Composite (for the Chinese market). The irregular components for these markets in these periods have been isolated and standardized. The differences between the irregular components for the markets concerned (in case the standardized irregular component is higher than 1 or lower than -1) and their sums are presented in Table 1. The irregular components for these markets are presented in Figures 1 and 2.

The interpretation of the research results

Calculated divergence factor amounts to 52,448 for NYSE/BSE pair and 55,273 for NYSE/SSE pair, so the factor is bigger for the NYSE/SSE pair than for the NYSE/BSE pair. Therefore, BSE is more convergent with NYSE than SSE in the period examined (the bigger the divergence factor, the lower the convergence).

Conclusions

This paper tried to shed light on time series decomposition which can be used for the measuring and comparison of the extent of the convergence of two or more time series (by using the divergence factors). A practical application of this method has been demonstrated using the example of time series consisting of the values of various selected stock market indices. Using the examples of the pairs of stock market indices (NYSE vs BSE and NYSE vs SSE) their convergence has been measured and compared using this method. The final conclusion was that NYSE in the period examined is more convergent with BSE than with SSE, since the divergence factor NYSE-BSE is lower than the divergence factor NYSE-SSE.

The empirical model developed above can be improved and extended in multiple ways. First of all, a larger dataset could be used to check whether the conclusions reached here remain valid or not. Second, alternative models might be used (instead of time series analysis) to study the long-run dynamics of price formation. It is worth to note that convergence (or divergence) can be measured using other quantitative methods, such as calculation of the correlation coefficients or one-way analysis of variance (one-way ANOVA). What cannot be provided by the alternative methods is the measurement of the impact of unexpected events on the data described by given time series (e.g. the impact of political or macroeconomic events on the capital market indices). These extensions are left for the future research.

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