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Number line estimation strategies used by children with dyscalculia and typically developing controls

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ABSTRACT

Aim
The aim of this study was to examine the effect of cognitive deficits in mathematical learning disabilities (such as dyscalculia risk) on mental number line processing with the use of the one-digit numbers as well as the symbolic and non-symbolic format of their presentation.

Method
We investigated number line estimation (NLE) in 20 children with mathematical learning disabilities (MLD) and 27 typically developing (TD) controls. They were examined with an NLE task using symbolic and non-symbolic numbers in the range of 1–9.

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Results
For all children, the greatest estimation error (EE) occurred for numbers located in the middle of the number line, but the effect was more pronounced in the MLD group. Moreover, both groups had a similar range for overestimation, but differed in the underestimation error. MLD children showed a greater left bias than the TD group with regard to nearly all numbers, except 7 and 8. An analysis of the EE for each number enabled us describe error distribution profiles and the probable estimation strategies used by MLD and TD groups.

Conclusion
It seems that the MLD group tends to assess the number line segments starting from the left-end benchmark, and setting an anchor in the center of the number line does not help them to estimate the positions of 4 and 6 correctly. In addition, all children had a greater EE for non-symbolic format, especially in the case of high magnitudes, which may be interpreted as a manifestation of both estimation and dot counting errors.

Keywords: dyscalculia, spatial-numerical association, mathematical abilities, mental number line, number line estimation.

INTRODUCTION

Dyscalculia is defined as a neurodevelopmental specific learning disorder leading to an impairment in mathematics (American Psychiatric Association, 2013) that manifests as difficulty in acquiring basic mathematical abilities. It is not explained by low intelligence, other developmental disorders, or motor and neurological disorders (Butterworth, Varma, & Laurillard, 2011; Kaufmann & von Aster, 2012). Dyscalculic children in comparison to normally achieving learners make more errors in counting, number naming, number comparison, and execution of arithmetical procedures and use immature strategies in mathematical tasks, such as verbal and finger counting (e.g. Geary et al., 2004; Mussolin, Mejias, & Noël, 2010). Representation of objects' numerosity is a basic capability in the development of arithmetic skills (Butterworth, 2010), yet children with dyscalculia have problems with elementary tasks such as enumeration of small sets of objects (Landerl et al., 2004), comparison of numerosity of two arrays of dots (Piazza et al., 2010), and retrieval of basic arithmetical facts (Geary, 1993). It is worth mentioning however that arithmetic fact retrieval may be considered as a result of deficient mental representation of numbers but also – a deficiency in the memory processing concerning the arithmetic facts retrieval (Geary, Bow, & Yao, 1992).

Numerical magnitudes have spatial representations, and this spatial grounding of numbers is based on the metaphor of the Mental Number Line (MNL) (Restle, 1970; Dehaene, 1997), which proposes that numbers are represented on a left-to-right continuum. Moreover, numbers and space are conjointly processed in the brain (Capeletti, Muggleton, Walsh, 2009; de Hevia, Vallar, & Girelli, 2008; Farnè, & Rossetti, 2006; Fischer & Shaki, 2014; Hubbard, Piazza, Pinel, & Dehaene, 2005; Göbel, Calabria, 2006; Sandrini & Rusconi, 2009). Importantly, the brain areas involved in this spatial-numerical processing reveal anatomical and functional abnormalities in dyscalculia (e.g. Isaacs et al., 2001; Molko et al., 2003; Mussolin et al., 2009; Rotzer et al., 2008; Rykhlevskaia et al., 2009).

Although the relationship between mathematical and spatial abilities (Mix, Cheng, 2012; Wai, Lubinski, & Benbow, 2009) seems to be unquestioned and widely documented in literature, there is less known about the direction of this relationship. LeFevre and colleagues (2013) reported in their longitudinal study that in primary school students there is a clear relationship between spatial, numeration (knowledge on number system) and calculation abilities and that the level of visuo-spatial skills predicted growth in performance of number line task, however not in the case of arithmetic or numeration abilities. Moreover, some findings show that typically developed and dyscalculic children do not differ in visuo-spatial working memory level, but the significant group differences appear after number line training (McCaskey et al., 2018; Michels, O'Gorman, & Kucian, 2018). On the other hand, some researchers (Kucian et al., 2011; Michels et al., 2018) showed that such training improved number line estimation in typically developed and dyscalculic children but did not have a beneficial effect in spatial working memory task both in children with dyscalculia and control group.

There are several types of Spatial Numerical Associations (SNAs, review by Patro et al., 2014; Cipora, Patro, & Nuerk, 2015) because numerical and spatial information can interact in many different ways. Importantly, not all SNAs correlate with arithmetic skills; even when correlated, this relation is not always evident or clear (e.g. Bonato, Fabbri, Umiltà, & Zorzi, 2007; Cipora & Nuerk, 2013; Georges, Hoffmann, & Schiltz, 2017; Hoffmann, Mussolin, Martin, & Schiltz, 2014; LeFevre et al., 2013). One type of SNAs is based on the representation of equidistant relations between numbers, and the ability to map numerical intervals to spatial ones. Experimental evidence and theoretical considerations indicate that during early childhood, such mapping firstly reflects a logarithmic pattern (the MNL is "compressed", Dehaene, 1997), showing a tendency to overestimate the magnitude of low numbers and to underestimate the magnitude of high numbers, while in later school years, the pattern changes to a linear one (Ashcraft & Moore, 2012; Booth & Siegler, 2006; Rouder & Geary, 2014). A typical task for measurement of this SNA category is the Number Line Estimation (NLE), in which participants are asked to assess the spatial position of a given number on an empty number line. In some variants the number line is flanked with the start and end numbers, typically 0 and 100 (or 1000). Alternative type of NLE task is to indicate the localization of a given number using the number line with a start number only (e.g. 0) and an additional hint about the length of number line which refers to particular number magnitude, e.g. "1" or "10". This reference length is usually displayed under empty number line (Link, Huber, Nuerk & Moeller, 2014). Traditionally, it has been assumed that the ability to position numbers reflects the shape of a numerical representation on MNL (compare with discussion below), and the fitting (linear or not) depends on the stage of cognitive development or math disabilities (Ashcraft & Moore, 2012). A logarithmic pattern of MNL typical for younger children has also been found in children with math disabilities (Geary, Hoard, Nugent, & Bailey, 2012). The linear representation manifested in precise NLE is interpreted as a clear predictor of prospective mathematical competencies (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Booth & Siegler, 2006; Geary, 2011).

However, this developmental log-to-linear representation shift is the one of theoretical accounts that have been questioned in the literature (see Cohen & Quinlan, 2018; Luwel, Peeters, Dierckx, Elke Sekeris, & Verschaffel, 2018; Sasanguie, Verschaffel, Reynvoet, Luwel, & 2016). Moreover, it was argued that logarithmic and linear response patterns in the NLE task provide no information about psychological representation of numbers (Cohen & Quinlan, 2018). It was proven that the performance of this task depends on application of strategies based on benchmarks. Moreover, particular strategies which utilize benchmarks can be measured both in children and adults when performing the NLE task. This alternative view focuses on the role of such strategies in NLE task performance (e.g. Barth & Palladino, 2001). For example, adults use a strategy based on proportion/subtraction to estimate the place of a number on a line i.e. by estimating the location of a given number as a proportion of the total length of the number line or by subtracting this number from the right end of the number line (Cohen & Blanc-Goldhammer, 2011; Hollands & Dyre, 2000). Such kind of NLE task performance was also reported in older children (Barth & Palladino, 2011). It is argued that the NLE response function based on this strategy is correlated with quantity representation. In contrast to the log-linear response pattern, the strategy based on benchmarks is modelled by the cyclic power model, which better fits the performance of a bounded NLE task (Cohen & Blanc-Goldhammer, 2011; Cohen & Quinlan, 2018). Namely, at first individuals rely on a beginning point (benchmark) and the simple power model explains the best the estimation response function. Later on, individuals estimate on the base of proportion using the beginning and end points, which means that children divide a whole number line into two half-sections. As a result, estimations are more accurate around the midpoint of the number line, whereas the location of numbers below the midpoint is overestimated and those above are underestimated. This response function can be explained by a one-cycle model. Finally, estimation is based on the midpoint and two intermediate reference points (placed at quartiles along the whole number line); in this case, the NLE data can be modeled by a two-cycle power model. Another problem that has been raised in the debate on the correlation between NLE performance and the development of quantity representation concerns the variability of tasks, stimuli, and analytical approaches in the studies that have examined this relationship (see Ebersbach, Luwel, & Verschaffel, 2013). These variations additionally complicate an already unclear picture. Consequently, it is difficult to compare results from the literature.

In summary, there exist vast research data concerning problems with counting, number comparison, numerosity estimation, or arithmetical fact retrieval in dyscalculics (see Butterworth, 2003, describing tools which are using for dyscalculia screening), yet the issue of NLE still seems to be insufficiently studied and explored (or at least broadly discussed), especially considering the distinct methods used for its examination, among other factors (see Ashcraft & Moore, 2012; Ebersbach et al., 2013). Another problem is that most studies on NLE performance use the number line flanked with the numbers 0 and 100 (or even 0 and 1000) as markers, yet there is no data concerning estimations of very basic one-digit number positions in children with mathematical learning disability (despite some attempts to develop such methods, like the one described e.g. by Cangoz, Altun, Olkun & Kacar, 2013; however, the authors did not report the results of the examination with the use of this tool). It is remarkable and surprising that previous studies have not focused on NLE performance in easy tasks using one-digit numbers and have only examined this issue for higher ranges of values. Moreover, in several studies, NLE tasks were tested with the use of paper-pencil method (or even e.g. one group was tested with a paper task, while the second was tested with a computer task – see Geary, Hoard, Nugen, & Byrd-Craven, 2008). Also, in most studies, only Arabic numbers were used as stimuli (what is justified by the use of 0–100 and 0–1000 number lines).

In our present research we attempted to answer the following question: may the impact of mathematical disability on NLE precision in children be observed in a very simple task? Specifically, we used a short number line limited by the numbers 1 and 9 and two number formats (symbolic and non-symbolic). The specific research questions were as follows: Can we find any (possibly subtle) differences between typically developing children and those with low math abilities in regard to the over- or underestimation of particular number values? What do these differences tell us about NLE performance in these two groups? Is performance dependent on number format? On the one hand, children with dyscalculia show severe deficits in symbolic number processing. On the other hand, they manifest a deficiency in object counting or estimation, which may be additionally related to deficit in visuo-spatial processing or working memory loading of set of dots.

There were several justifications for the choice of the 1–9 number line. First, we were interested in testing whether differences between the two examined groups would appear upon considering only short-range one-digit numbers. Second, it would be problematic for participants to count dots ranging from 0 to 100 elements (e.g., to correctly count 45, 77, or 98 dots in order to locate the corresponding number on the line). In particular, the use of a numerical interval of 1–9 enables to avoid the need to count a large number of dots (in the 0–100 range) and sheds some light on potential differences dependent on format factor. Third, for two reasons, we did not use the numbers “0” and “10,” which have been employed in previous studies with short number lines. It seems indisputable that children in this age manifest an established representation of zero (and empty sets, “no objects”, “nothing”) and know how to use it in both symbolic and nonsymbolic format (e.g. Krajcsi et al., 2017; Merrit & Brannon, 2013; Wellman & Miller, 1986). Moreover, mental representation of number “0” has its neural correlates in parietofrontal network (Rinaldi & Girelli, 2016). In our present study, however, number “0” could be problematic in the task using non-symbolic format – lack of dots in this type of task would result in an empty screen only with a number line displayed on it. In short, trials with „0” number represented by empty screen might be confusing for children. On the other hand, the application of the 1–10 number line would result in the lack of clear middle point. In our task, the central point was assigned to “5.” However, on a 1–10 line, it would be placed between "5" and "6." Finally, by using a small range of numbers, we were able to analyze and discuss the estimation errors for each number in detail.

We predicted that, even in such a simple NLE task, the estimation error will be generally greater in the group of children with low math abilities than in typically developing children, despite the fact that the NLE responses probably follow a linear pattern in both group of children at this age examined in our study (see Geary et al., 2008). Moreover, we assumed that differences would be present in the NLE performance of two tasks (with the use of symbolic and non-symbolic numbers), because of additional counting problems in the group of children with mathematical learning disabilities. Finally, the results of present study might provide some additional noteworthy data concerning the processing of one-digit-number line in children with dyscalculia risk in comparison to typically developed ones. The differences in NLE error between both groups, which are of our interest in the present study, may bear a significant contribution to the previous knowledge, especially in the context of a disputable idea that was recently proposed by Sella, Sasanguie and Reynvoet (2020). They examined a determination of ordering of 1–9 digits presented in triplets (e.g. 7–8–9) and on the base of obtained results they concluded that symbolic order processing in the case of small number magnitudes seemed to be not the manifestation of MNL development but rather the effect of familiarity related to everyday-learned triplets of numbers (like 1–2–3 or 3–6–9) stored in a long-term memory. Although their study was performed on young adults, it may suggest that similar pattern should be observed also in children, both dyscalculic and typically developed, because all of them are familiarized with such triplets as these used by Sella et al. For this reason we did not expect any significant differences in 1–9 MNL processing between the two groups of children participating in our study.

METHOD

Participants

Forty-seven children (24 girls and 23 boys, mean age = 13.6; ranged from 9.83 to 16.58) participated in the study. Twenty had difficulties in mathematics (11 girls and 9 boys, mean age = 13.36 and 12.96, respectively), and 27 children (13 girls and 14 boys, mean age = 14.96 and 12.95, respectively) had normal achievement in mathematics (control group). We classified the first group as the mathematical learning disability group (MLD; similar to Geary et al., 2008) and the second as the typically developing group (TD). The mean age of MLD and TD children was 13.2 and 12.9 years, respectively. The mean ages did not differ significantly between the MLD and the control group ($t(45) = 1.23$; $p > .05$), nor did the number of boys and girls ($\chi^2(1) = 0.21$; $p > .05$).

Participants were students of the elementary schools from Pomeranian and Kujavian-Pomeranian Voivodeships in Poland. They all received normal education in mathematics. Participants were placed in the MLD group by psychologists in psychological clinics, based on their standard diagnostic criteria, including, e.g., general intelligence within a normal range, low score (under 8 points) in the Arithmetic subtest of the Wechsler Intelligence Scale for Children-Revised (WISC-R; Wechsler, 1974), the level of mathematical achievement at school, and the psychologist's observations. Importantly, the most of the MLD participants had not diagnosis of dyscalculia, because in Poland this deficit may be diagnosed quite late, at the end of primary school. Thus, the majority of participants got diagnosis of dyscalculia risk. This is why here they are described as mathematical learning disabilities group, instead of dyscalculia group. The parents of all participants gave their informed consent to participate. The procedures used in the study were approved by the local Ethics Committee in Collegium Medicum, Nicolaus Copernicus University in Toruń (number of decision: KB 99/2018). All participants were healthy, had no history of neurological problems, and had normal or corrected-to-normal vision.

Tasks and stimuli

The study procedure was divided into two tasks: one with the use of symbolic numbers and another with the use of non-symbolic numbers to be placed along a number line. The order of tasks was not randomized (each participant performed the task with the symbolic format first and then the one with the non-symbolic format). Each stimulus (see Figure 1) consisted of a horizontal number line without ticks marking integer numbers (except the start and end points, which were labelled "1" and "9," respectively). The line was half of the screen width, i.e., 960 px on a 1920 × 1080 display. The number line was horizontally aligned to the center of the screen and located at 3/4 (from the top) of the screen height. For each stimulus, an arrow (with a height equal to 1/9 of the screen height) was presented directly above the line and pointed to a location that was determined by the participant. Above the line and the arrow, the given numbers (in the form of an Arabic digit in the first task and a set of dots in the second) were displayed at 4/10 (from the top) of the screen height and at the middle of the line. In each trial, the number indicated above the line ranged from 1–9 and was displayed in white (RGB 230, 230, 230). All stimuli were presented on a black background (RGB 25, 25, 25). Contrast of both colors was reduced to avoid eye fatigue and afterimage effects.

Figure. 1. Examples of stimuli used in the tasks.

In the task using symbolic number format, the height of each digit in the stimulus was 222 px, and the width was between 152 and 200 px, depending on the number value (e.g., 152 px for digit 1). In the task using non-symbolic number format, each stimulus consisted of a set of dots. The diameters and locations of the dots were selected to be the same in all experimental repetitions with respect to the average total area of dots, the area and perimeter of the convex hull around the dots, density (ratio of dots' area per convex hull area), the dots' center, and the center-weighted dot size; differences in the assumed parameters were not larger than 2% (c.f. Gebuis & Reynvoet, 2011). Examples of the stimuli are illustrated in Figure 1. The order of tasks was the same for all participants, but the order of numbers within the task was randomized. Each number used in the task was repeated four times (that gives 36 trials in the task and 72 trials in the whole procedure). In each trial, participants were asked to estimate the location of a number on the presented number line (number-to-position NLE task) by clicking the right number position with the use of left key of a standard computer mouse. There was no time limit for performing the task, so children were instructed to estimate the location of a given number as precisely as he or she could, with no time pressure. The estimation error was calculated as the ratio of the absolute number of pixels between the clicked position and the correct position to the length of the whole number line (also measured in pixels) and was expressed as a percentage. Each task consisted of 36 trials (each number was repeated 4 times), and the order of the trials was randomized.

Although in the previous studies on NLE participants did not estimate the position of the numbers displayed on the number line (i.e., 1 and 9), it has to be underlined that in those reports NLE tasks contained only the symbolic (Arabic) format of numbers. This is why in our study participants were asked to indicate the position of all numbers from 1 to 9, despite the fact, that both 1 and 9 were visible at the ends of number line. Although in the case of task with Arabic numbers displayed above the line, it does not seem to make any sense, in the task with dots displayed above the line children were required at first to count or subitize (in case of small numbers the number of objects can be determined fast and at glance, without counting process) the dots and then to place this number represented by dots on the line. Therefore, we could assume that any eventual estimation errors which could be observed for number 9 in the task with dots would be rather an effect of errors in dot counting or relying on quantity estimation ability. In order to assure that the two tasks were identical but different only in terms of the format of numbers, we asked participants to estimate positions of 1 and 9 in both tasks (i.e., Arabic numbers and dots). This allowed us to examine the impact of the format of numbers on the task performance.

Apparatus and software

The participants were comfortably seated at about 60 cm in front of a portable computer monitor. The stimuli were presented on a 13.3” LCD screen with a resolution of 1920 × 1080 px and a 60 Hz refresh rate. The participants responded by left-clicking on a standard computer mouse. The stimulus presentation and the recording of participants' responses were controlled by the computer test Prokalkulia 6–9, designed and implemented by our team (Gut, Goraczewski & Matulewski, 2016). It was prepared in the Microsoft Visual Studio 2013 Enterprise integrated development environment with the use of C# language. It requires .NET Framework 4 Client Profile and Windows 7 or higher.

Statistical analysis

Because of the unequal group size, non-normal data distribution, and the presence of outlier data points, we employed robust statistical methods to compare the MLD and TD groups (Field, Miles, & Field, 2012; Field & Wilcox, 2017). Robust mixed-effect 2×2 analyses of variance (ANOVAs) were calculated with the 20% trimmed means to investigate general group differences in estimation error, underestimation, and overestimation on the number line between groups (Wilcox, 2012). Group (MLD/TD) was considered as the between-subjects variable, and number format (symbolic/non-symbolic) as the within-subjects variable. The 20% trimmed mean achieves similar power to the mean calculated from a normally distributed sample; when outliers are present, it also has smaller standard error (Mair & Wilcox, 2016). To investigate the group differences at specific magnitude intervals (numbers 2 and 3 defined as low, 4–6 as middle, and 7 and 8 as high), we employed robust 2×2×3 ANOVAs on the 20% trimmed means considering the additional factor of magnitude level (low, middle, or high). Analogically, group differences in specific magnitude were tested using robust 2×2×9 ANOVAs on the 20% trimmed means, considering the additional factor of number magnitude (1–9). For significant interaction effects, we performed robust pairwise comparisons based on M-estimators and 2000-sample bootstrapping using the family discovery rate (FDR) correction for multiple comparisons (Benjamini, Hochberg, 1995). All statistical analyses were performed in R using packages WRS2 for robust ANOVAs and rcompanion for robust pairwise comparisons (Mair & Wilcox, 2016).

RESULTS

Effect of factors on estimation error value

To examine the effect of group (MLD vs. TD) and format of displayed numbers (symbolic vs. non-symbolic) on mean Estimation Error (EE), the data were submitted to a robust two-way mixed ANOVA. Data obtained in trials for numbers 1 and 9 were excluded from these analyses because both numbers were displayed on the screen as the left- and right-end points on the number line. Thus, we assumed small EE for these numbers as a result of their explicit presentation on the number line, and consequently, that the EE for 1 and 9 would understate the mean values calculated for all numbers.

We used group and format (2×2) as the between-subjects factor (the group variable) and the within-subject factor (the number format variable) and the EE as the dependent variable. The analysis revealed that there was a significant effect of group, $F(1, 45) = 12.29$, $p < .01$, with a significantly greater EE for MLD children (4.86%) than TD (control) children (3.49%). There was also a significant main effect of number format, $F(1, 45) = 7.11$, $p < .001$, with a smaller EE in the task using symbolic number format (3.84%) than in the task using non-symbolic format (4.51%). No significant interaction was found between these two factors. However, the additional calculation of differences between groups separately for the symbolic and non-symbolic format of numbers showed that, in the case of non-symbolic format, the mean EE was smaller in the control group than in the MLD group ($p < .01$), and the same effect was revealed in the case of symbolic format ($p < .05$).

To examine the effects of group, format of displayed number, and number interval: low (2 and 3), middle (4, 5, and 6), and high magnitude (7 and 8) on mean EE, the data were submitted to a robust three-way ANOVA. We used group (2), format (2), and interval (3) as the within-subject factors and the EE as the dependent variable. The analysis revealed that group exhibited a significant effect, $F(1, 45) = 37.09$, $p < .001$, with significantly greater EE observed in MLD children than in control (TD) children (as revealed in the two-way analysis of variance reported above). Number format also had a significant main effect, $F(1, 45) = 5.76$, $p < .05$, with a smaller EE in the task using symbolic number format than in the task using non-symbolic number format (as shown in two-factor analysis of variance described above). In addition, number interval exerted a significant main effect, $F(2, 90) = 58.22$, $p < .01$ on dependent variable. The comparison of the mean EE for each number interval (low, middle, and high magnitude) showed significant differences in mean EE between the low (3.61%) and middle magnitude intervals (6.33%, $p < .001$), between the middle and high magnitude intervals (4.42%, $p < .001$), and between the low and high magnitude intervals ($p < .01$). Moreover, we found significant interaction between group and number interval, $F(2, 90) = 7.56$, $p < .05$ (see figure 2) and between number format and number interval, $F(2, 90) = 10.03$, $p < .05$; (see figure 3).

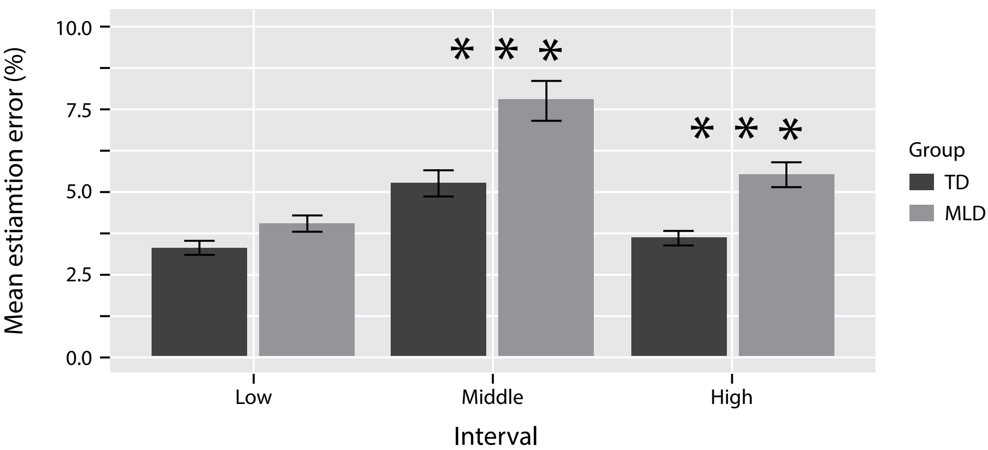

Figure 2. The interaction between group and number magnitude interval with the mean estimation error (EE; mean values represented by bars). The error bars represent the standard errors of the mean (SEM) values.

As presented in figure 2, for middle magnitude numbers, we observed a bigger EE in MLD children than in TD children ($p < .001$); the same effect occured for high magnitude numbers ($p < .001$). The difference between groups with respect to low numbers was nonsignificant.

As illustrated in figure 3, for the symbolic number format, comparisons revealed significant differences between the EE for low magnitude numbers (3.78%) and middle magnitude numbers (6%, $p < .001$), as well as between the EE for middle and high magnitude numbers (3.94%, $p < .01$). The difference between EE for low and for high numbers was nonsignificant. For non-symbolic format, significant differences were found in EE for low (3.56%) vs. middle (7.03%) numbers (p < .001), between EE for middle vs. high (5.18%) numbers ($p < .05$), and between EE for low vs. high numbers ($p < .01$). None of the remaining interactions reached significance.

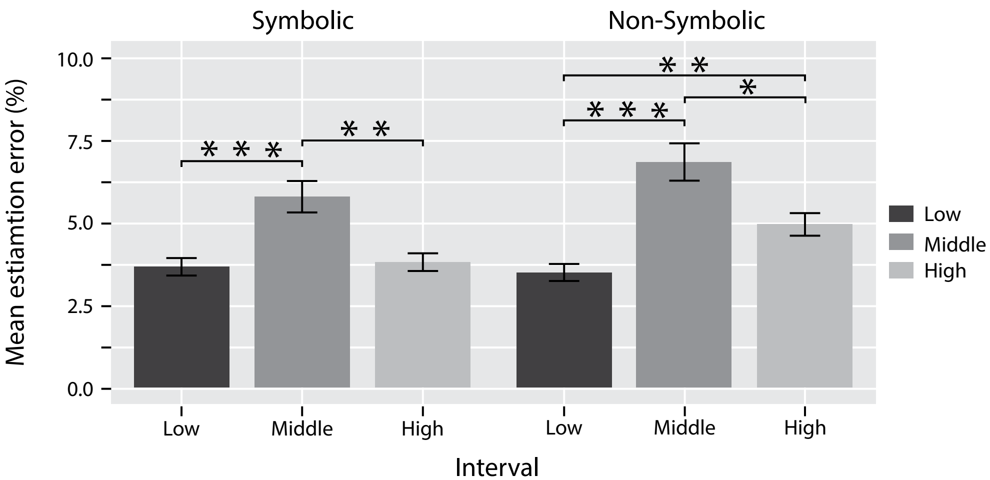

Figure 3. The interaction between number format and number magnitude interval with the mean estimation error (EE; mean values represented by bars). The error bars represent the standard errors of the mean (SEM) values.

Effect of factors on underestimation and overestimation values

To examine the effects of group and format of displayed number on the mean underestimation error (UE), the data were submitted a robust two-way mixed ANOVA. We used group and format (2×2) as the between-subjects and the within-subject factors and the UE as the dependent variable. The analysis revealed a significant effect of group, $F(1, 45) = 13.78$, $p < .01$, with significantly greater UE observed in MLD children (5.84%) than in control (TD) children (4.02%). No significant main effect of format and no significant interaction between these two factors were found.

To examine the effect of group, format, and number interval on mean UE, the data were submitted to a robust three-way ANOVA. We used group as the between-subjects factor, format and interval as the within-subjects factors ($2 \times 2 \times 3$)

and the UE as the dependent variable. The analysis revealed a significant effect of group, $F(1, 45) = 34.76$, $p < .001$, with significantly greater UE observed in the MLD than in the TD group (as revealed in the robust two-way ANOVA reported above). A significant main effect was also found for number interval, $F(2, 90) = 51.94$, $p < .01$. The comparison of UEs for low, middle, and high numbers revealed significant differences in mean UE between the low (3.15%) vs. middle (6.51%) magnitude interval ($p < .001$), between the middle vs. high (4.62%) magnitude interval ($p < 0.001$), and between the low vs. high magnitude interval ($p < .01$). None of the interactions between these factors reached significance.

We performed a similar analysis for the overestimation error (OE) data. First, we used group and format (2 × 2) as the between-subjects and within-subject factors and the OEs as the dependent variable. The analysis revealed a significant effect of format, $F(1, 45) = 5.57$, $p < .05$, but no significant main group effect nor interaction between group and format were found.

Estimation Error for each of number magnitude (1–9)

To examine the effect of group, format, and magnitude of displayed number (from 1 to 9) on mean EE, the data were submitted to a robust three-way ANOVA. We used group, format, and magnitude (2 × 2 × 9) as the between-subjects and the within-subject factors and the EE as the dependent variable. The analysis once again revealed that group had a significant effect, $F(1, 45) = 46.57$, $p < .001$, with greater EE observed in the MLD than in the TD group. Format also had a significant effect, $F(1, 45) = 4.56$, $p < .05$, with bigger EE observed for non-symbolic format. A significant main effect was also found for number magnitude, $F(8, 360) = 1398.57$, $p < .01$ and for interaction between group and number magnitude, $F(8, 360) = 67.92$, $p < .01$. Figure 4 presents this interaction: The differences between the EE of the MLD and the TD children were obtained for each

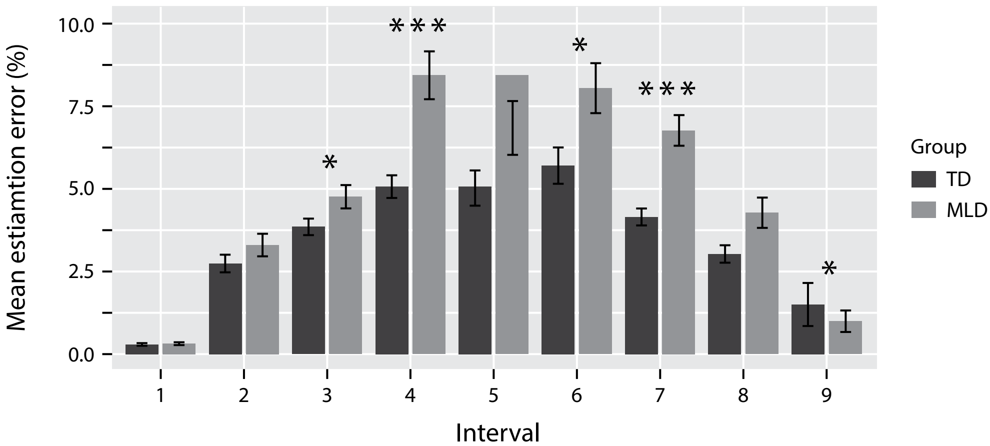

Figure 4. The interaction between group and number magnitude (1–9) in the mean estimation error (EE; mean values represented by bars). The error bars represent the SEM values.

of the 9 magnitudes. Significant group differences were observed for number 3 ($p < .05$), number 4 ($p < .001$), number 6 ($p < .05$), number 7 ($p < .001$), and number 9 ($p < .05$). The mean estimation error was significantly greater in MLD than TD group in almost all of these number magnitudes, except number 9, where the EE was greater in TD group.

Additionally, because of the significance of the format factor, we compared the EE of the groups for particular number magnitudes, separately for the task with symbolic number format and separately for the task with non-symbolic number format, as illustrated in figure 5. (A and B, respectively). For the symbolic number format, significant group differences were observed for number 4 ($p < .01$), number 6 ($p < .05$), and number 7 ($p < .05$). For the non-symbolic number format, a significant difference was observed only for number 7 ($p < .001$).

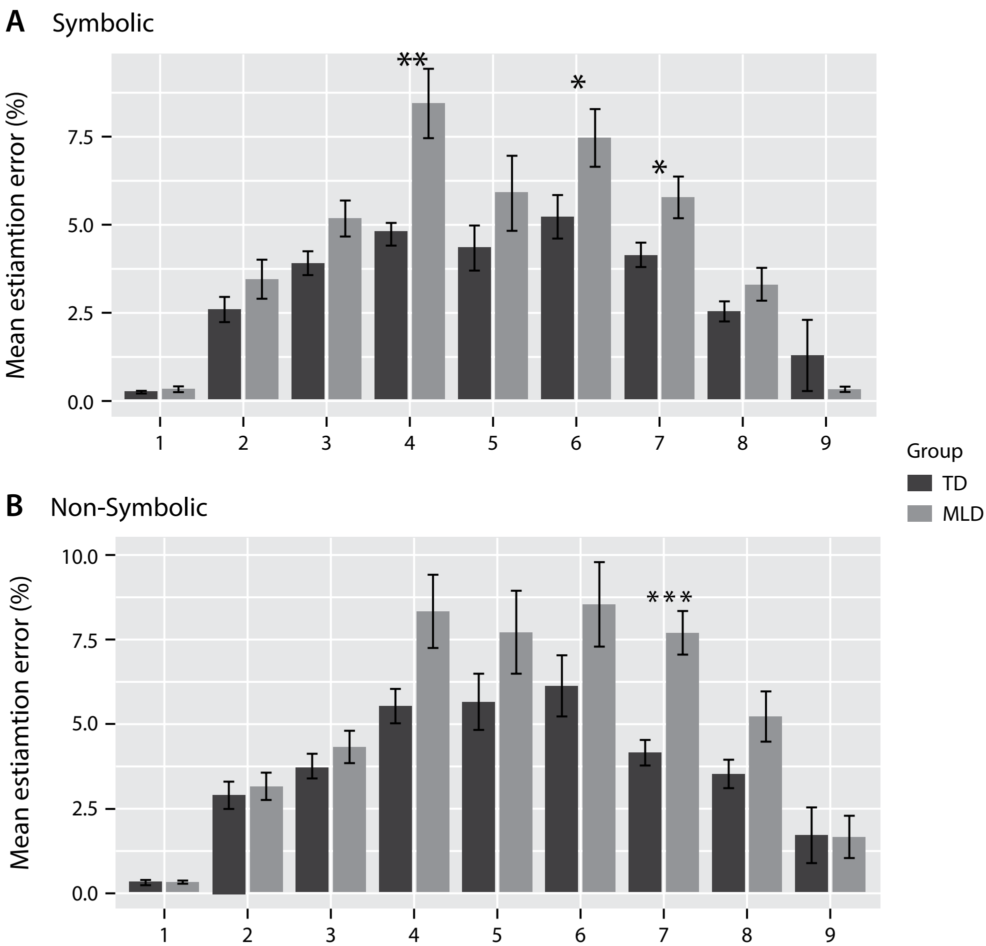

Figure 5. The group differences for each number magnitude in the mean estimation error, calculated separately for symbolic (A) and non-symbolic (B) format of stimulus (EE; mean values represented by bars). The error bars represent the SEM values.

Underestimation and overestimation for each number magnitude (1–9)

We were interested whether the problem of estimation resulted mainly from under- or overestimation (do MLD children predominantly under- or overestimate the spatial position of one-digit numbers?) as well as whether it depended on the number magnitude or not. Thus, we performed similar analyses (i.e., analogous to analyses with EE described above) using UE and OE as the dependent variables.

First, we used group as the between-subjects factor, format and magnitude as the within-subject factors (2 × 3 × 9) and the UE as the dependent variable. The analysis revealed a significant effect for group, $F(1, 45) = 44.78$, $p < .001$, with greater UE observed in the MLD than in the TD group, as well as for number magnitude, $F(1, 45) = 609.7$, $p < .01$, and for the interaction between group and number magnitude, $F(8, 360) = 50.31$, $p < .01$. Figure 6 (lower part) presents this interaction and the differences between the UEs in the MLD and TD groups, obtained for each of the nine number magnitudes. Significant group differences were obtained for number 1 ($p < .001$), number 2 ($p < .01$), number 3 ($p < .05$), number 4 ($p < .001$), number 5 ($p < .05$), number 6 ($p < .05$), and number 9 ($p < .05$).

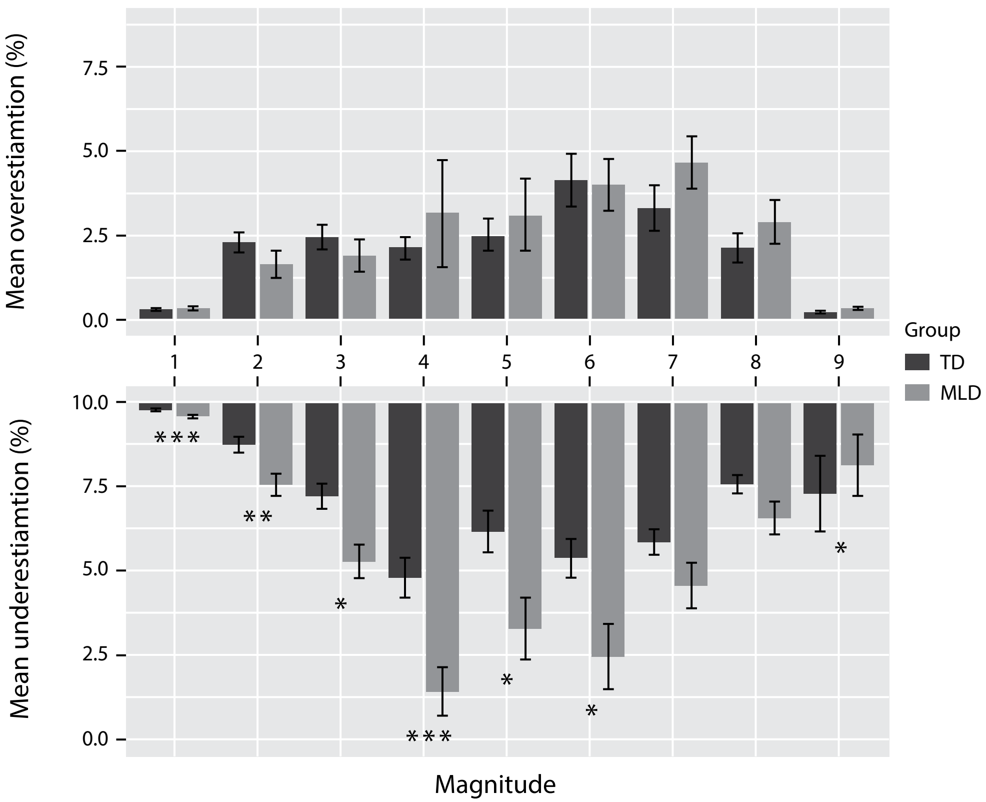

Figure 6. The group differences for each number magnitude in the mean overestimation error (upper part) and underestimation error (lower part), (OE and UE; mean values represented by bars). The error bars represent the SEM values

Second, we performed a similar analysis for OE data. We used group as the between-subjects factor, format and magnitude as the within-subject factors ($2 \times 2 \times 9$) and the OE as the dependent variable. The analysis revealed a significant effect only for magnitude, $F(8, 360) = 162.26$, $p < .01$. Neither the remaining main effects nor interactions between these factors reached significance level. Figure 6 (upper part) presents separately the mean and SEM values obtained in groups for each number magnitude. No group differences were statistically significant.

DISCUSSION

The aim of the present study was to investigate the effect of low numeracy skills on processing the spatial representation of numbers during a Number Line Estimation (NLE) task in children. We considered the estimation error (EE) and the separate effects of over- and underestimation biases. Additionally, as the NLE task was implemented using both symbolic and non-symbolic number formats, we were able to determine the impact of number format on EE.

Effect of low mathematical abilities and number format on general NLE performance

In terms of EE, a generally poor performance was recorded for children with low numeracy skills (MLD group) in comparison to the TD control group, as expected. This result is in accordance with other results reported in previous studies (e.g., Geary et al., 2008; Kucian et al., 2011). However, in addition to the indisputable difference between the two groups of children, we also obtained an effect for number format, with bigger EE in the task for numbers presented as a set of dots for all children. The underestimation marker calculated for each magnitude and format (see discussion of this result below) implies that the greater EE for dots is a consequence of counting errors found in MLD children and a consequence of carelessness in TD children. Namely, TD children also made errors in identification of a number presented on the screen, however in their case the errors did not result from the deficit of counting ability, but rather were caused by the fact that TD children did not count. Probably, they based on a number estimation and subitizing, which led to errors in the case of high magnitude numbers. Thus, in the MLD group, the estimation errors could result from the deficit typical of dyscalculia, as dyscalculic children typically manifest counting problems in small sets of objects, which was confirmed by our observations. Meanwhile, in the TD group, this could be the effect of carelessness in performance, i.e., TD children wanted to finish the task as fast as possible so they did not count dots in the sets of 7, 8 and 9, but they rather based on estimation of quantity of dots. Moreover, it is quite likely that both carelessness and estimation of quantity (instead of counting higher number of dots) stemmed from boredom, because for typically developing children such type of task may be considered too easy.

What do we observe when focusing separately on the effects of group and format on the underestimation and overestimation values? In the case of underestimation, both the effect of group (greater left bias in the MLD group) and the effect of number format (the same as described above for EE) were observed. Interestingly, none of these factors affected the values of overestimation. The right-side bias was not dependent on mathematical disability nor the number format. All children overestimated the location of the presented numbers, but this error did not apply in particular to participants with MLD, as revealed by EE values. Namely, in case of overestimation there are no significant group differences. Thus, in both MLD and TD groups, the overestimation error is the same.

Effect of math learning disability, number format and number magnitude interval on NLE performance

An examination of the results concerning the effects of group, format, and number magnitude provided us with a wider perspective of the NLE task performance in TD children and in those with MLD. Taking into consideration the EE values calculated for three magnitude intervals (low numbers except for "1," middle numbers, and high numbers except for "9"), the analysis showed that children in both groups demonstrated the highest error values in estimating the location of numbers in the middle magnitude interval, i.e., the numbers "4," "5," and "6," when displayed in either symbolic or non-symbolic format. This effect may be due to the anchoring of attention around the numbers "1" and "9," which flanked the number line used in the task. It could also result from the effect of locating both low and high numbers by estimating from the left (in case of "2" and "3") and the right (in case of "7" and "8") end of the line, which is probably easier. The same magnitude-interval effect was observed for underestimation but not for overestimation, as illustrated by the distribution of the UE and OE values calculated for each number from "1" to "9" (see discussion below).

As expected, the interaction between number magnitude interval and group showed a very clear difference in the EE obtained for the middle number interval in both groups, with significantly poorer estimates performed by MLD children. Thus, the greatest error in NLE within the middle magnitude interval may stem from poor precision. Also, with respect to this number magnitude interval in the MLD group, we observed the greatest variance in EEs, manifested as the highest standard mean errors (SEMs). A significant group difference was also obtained for the high magnitude interval; however, EE values smaller than these obtained for the middle magnitude interval were found in both the TD and MLD groups. This difference may be interpreted as the manifestation of problems with NLE in the latter group, but also as manifestation of problems with counting of dots during the trials using non-symbolic number format (in particular, for the numbers "7" and "8"; see the discussion concerning the particular number magnitudes and formats below). Finally, the smallest and most similar EE were associated with low magnitude numbers, which is a sign of proficiency in small number processing in all children. This is likely a demonstration of the widely reported greater ease and automaticity in the processing of small numbers. For example, previous studies uncovered the shift in attention toward small numbers (e.g., Cai, Li, 2015) as well as their prevalence in everyday experience (e.g., Dehaene et al., 1993, but see review by Nuerk, Moeller, Klein, Willmes, & Fischer, 2011, who provided examples of more frequent everyday experiences of dealing with multi-digit numbers). In addition, the way in which low-magnitude numbers guide our attention was also proven in different studies, which demonstrated: the attentional bias toward the left side of the MNL in a numerical interval bisection task (Göbel et al., 2006; Longo & Lourenco, 2007; Longo, Lourenco, & Francisco, 2012), the advantage of low-magnitude numbers in a study on temporal perception of digits (Schwarz & Eiselt, 2009), the ability of small numbers to direct attentional focus using a target detection task (Cai & Li, 2015), and the more frequent production of low-magnitude numbers than high-magnitude numbers in a random number generation task (Boland & Hutchinson, 2000; Loetscher & Brugger, 2007). All these data confirm that small numbers are processed faster.

However, the small values of EE for the numbers "2" and "3" may be also linked to a well-known inborn capability called subitizing (review by Dehaene, Molko, Cohen, & Wilson, 2004; Feigenson, Dehaene, & Spelke, 2004; Patro et al., 2014), which is manifested as the fast and effortless determination of numbers for a small set of objects. Subitizing also refers to the easier processing of low magnitude numbers, here specifically in context of effortless processing of one, two, three or four dots presented above the number line. We think that subitizing could be a reason of the smallest EE values in the case of low numbers, which means that the cause of estimation error for low numbers (which can be subitized) is not the same as in case of high number of dots (7 and 8). In case of high numbers that is the counting problem or the quantity estimation errors. In other words, it is unlikely to do estimation error for 2 or 3 dots because of the problem with counting such small set of objects, which are always subitized by individuals.

Differences obtained for each number magnitude

The most enlightening findings were revealed upon considering particular numbers, which enabled us to conjecture about the probable strategies of number position estimation used by TD and MLD children. What emerged from the group differences calculated for each number format and for each magnitude? The analyses showed again the same main effects for group and format as those described above. However, the main effect of number magnitude and the interaction between group and number magnitude were also significant. The EE, UE, and OE values for each number allowed us to explore the differences within the three number intervals in greater detail and to reveal several important findings in the case of numbers "1" and "9," which were displayed to the participants on the number line.

First, we found that the greatest EE value, which was obtained for the middle number interval, is not related to the difficulty in locating the number "5," but rather "4" and "6." The group differences for these two numbers were significant and showed greater EE in the MLD than TD children. For the number "5," the EE values were smaller, and the difference between groups was not significant. This suggests that all children (independently of their abilities in mathematics) set a benchmark more or less in the center of a given number line to indicate the position of this number. Moreover, in case of TD children, the differences between groups obtained for "4" and "6" suggest that the anchor placed in the position of "5" is more useful for estimating the position of numbers adjacent to "5." Meanwhile, in the MLD group, despite the existence of a relatively small EE for positioning the center number, the errors for numbers next to "5" (on the left and right) were the highest as compared to all other mean EE values found for the other magnitudes. Another important result concerning this magnitude interval was that the higher variance was found in this group for "4", "5," and "6". This result suggests that children with low numeracy skills (MLD) did not face the same difficulty in locating the position of these numbers. This finding appears to be consistent with expert opinions and reports that describe dyscalculia as a very complex deficit and point that the population of children suffering from low mathematical abilities is very heterogeneous (review by Kaufmann & von Aster, 2012).

Second, an even more noteworthy result concerns the differences between groups found for the numbers "3" and "7." In both cases, the EE was bigger in the MLD group, particularly in the latter case. These two results seem to stem from the difficulty in estimating number positions located at a distance from the end of number line (either left or right, respectively) and from the center point (number "5"). However, an additional exploration of the error values calculated for the symbolic and non-symbolic format revealed that, in case of "7," there is one additional cause identified for the poorer estimation performed by MLD children as well as the general high EE value for all children. The differences between groups were significant for both symbolic and non-symbolic format, but apparently greater EE was found in the MLD group for the non-symbolic format. This suggests that additional errors were due to problems in the counting of dots and corroborates the results of previous studies showing that children with dyscalculia manifest disability in the counting of objects (Koontz & Berch, 1996) or in the comparison of numbers presented in non-symbolic format (Landerl, Fussenegger, Moll, & Willburger, 2009; Mussolin et al., 2010). Such problem probably does not exist for number "3" because of subitizing which reduces chances of counting errors. Thus, whether the NLE task can be properly performed with dots instead of digits may be questioned because, in fact, this task measures NLE affected by counting dis(abilities).

An inspection of the patterns in the results for the under- and overestimation values, as well as for the symbolic and non-symbolic numbers, allowed us to explain high error values found for high-magnitude numbers. For high numbers of dots, participants produced more errors because of mistakes (e.g., confusing number "8" with the number "9" or "7" with "8"); the position of "8" was often shifted from the correct position of "7." Thus, this right-side bias is caused not only by poor MNL processing but also because the number was determined incorrectly. These counting errors may be also caused by participant's carelessness in counting, which was observed in both TD and MLD children. Although participants were asked to perform the task as precisely as possible, some of them probably tried to finish it as fast as possible. Some children determined number of dots "at a glance." As a result, in the case of nine dots, they marked its location in the place where they estimated the location of number "8" (not because of difficulties in NLE but because of errors in dot counting). This was also confirmed by the interaction between the number magnitude interval and the number format. As observed, in the case of Arabic digits there was no difference between EE values for low and high number intervals. In turn, in the case of dots EE were bigger for high numbers than for low numbers. In other words, when children were asked to estimate the location of a small or high number presented as a digit, the value of EE was similarly low, but the necessity to count objects led to higher EE for high numbers.

Finally, we revealed the difference in EE between groups obtained for the number "9," which is in absolute contradiction to the results concerning the rest of the number magnitudes. A relatively high value of EE was obtained for "9" despite this number being shown on the screen as an anchor at the end of the number line. This finding is quite incomprehensible, but what is more, EE was significantly larger in the control TD group. The explanation of this finding may be twofold, and we can find some hints in the pattern of the results concerning the errors calculated separately for symbolic and non-symbolic numbers, as well as errors calculated separately for under- and overestimation. First, the estimation errors for locating "9" were visible only in underestimation values (there was almost no overestimation at all). This may be interpreted in the context of similar underestimations which were revealed in the case of other high-magnitude numbers, especially those presented in non-symbolic format. A further justification for this interpretation comes from the EE obtained in the task with non-symbolic number format. Quite large EE values were found for both groups which is likely an effect of incorrect number determination. Second, in the case of symbolic format, there was relatively high value of EE observed in TD group, while almost no error was observed in MLD children. However, it is worth noting that the EE variance in the control group is sizeable, possibly suggesting that the estimation error for this particular condition (i.e., for "9" presented in the symbolic format) stems from the imprecision of task performance demonstrated by several children in this group. In addition, children learn to count to 10, not to 9; therefore, they tend to associate 9 with a position before the end point; if this is the case, the effect should vanish for lines numbered from 1–10. However, this would also break the symmetry, as the middle point of this line would then be between 5 and 6.

An overall view of the over- and underestimation values for particular numbers indicates that children tend to underestimate the location of numbers more than overestimate it and that this underestimation is particularly observed in children with disability in mathematics. The mean OE values were smaller than the UE values, and no group differences were found in the right-side bias. The differences between groups with respect to underestimation concern most of number magnitudes. It seems that MLD children shift the location of numbers to the left end of the number line. This is probably the manifestation of certain strategy used during NLE performance, i.e., strategy based on the "measurement" of distances from number localizations, and from the beginning of the number line, without using additional benchmarks. It is also a reasonable to think that the shift in spatial attention evoked by low-magnitude numbers (discussed above in the context of number magnitude effect) is more effective in individuals with MLD. The validity of this interpretation may be verified, e.g., in experimental procedures examining the orienting attention shift, using, for example, small vs. high numbers preceding a stimuli presentation or random number generation task performed by TD and MLD participants.

Limitations of the study

The current investigation provided some valuable findings, but it is not free of shortcomings, which should be considered in the further studies on similar topic. First, it may be difficult to compare the results of current study with the data reported previously in the literature. Among others, we could not highlight a clear overestimation for the location of high-magnitude numbers vs. low-magnitude numbers to verify whether the pattern of number placement is typical of that described in the literature for TD children because we used a number line that was shorter than those used by other researchers for this purpose. In previous studies, researchers used a wider range of numbers in the NLE task, typically from "0" to "100" (e.g., Kucian et al., 2011; Rouder, Geary, 2014) or even from "0" to "1000" (e.g., Ashcraft & Moore, 2012). In one study, a shorter number line (from "1" to "10") was used for much younger participants (e.g., Berteletti et al., 2010). This issue, however, was addressed in the Methods section. Another limitation, at least partly related to the abovementioned one, is the problem with maintaining the motivation of participants. The performance of a task as easy as NLE using numbers in the range of "1" to "9" could be wearisome and boring, especially for control (TD) children. It is possible that, in some cases, the EE were the result of fatigue or boredom. In future studies on this issue, it would be reasonable to implement tasks that differ in difficulty or to narrow the age range of participants. Moreover, it would be valuable to consider the use of two variants of NLE tasks: bouded and unbouded ones, since they could provide some important data concerning NLE strategies, which are different in case of each type of task: bounded and unbounded. It seems important especially in the case of participants aged like in current study (see Link et al., 2014).

Conclusion

Computer NLE tasks using symbolic and non-symbolic number format were implemented in this study in order to examine the capacity to estimate one-digit number location in age-matched children with low numeracy skills (MLD) and typically developing children (TD) in terms of estimation error as well as under- and overestimation values. Detailed data were collected, showing that all children revealed the greatest EE for numbers located in the middle of the number line, but the effect was stronger in the MLD group. This group effect contradicts conclusions drawn by Sella et al. (2020). Moreover, the groups showed a similar range for the OE but differed in terms of the UE. All children manifested a tendency of left-side bias, but it was more visible in participants with low numeracy skills. These children showed a greater left bias than the control group for most number magnitudes. The exploration of EE for each number allowed for the detection of error distribution profiles, which suggests probable strategies used by the MLD and TD groups for estimating the location of numbers on a number line. It seems that children with low mathematical abilities tend to assess the number line segments starting from the left-end point, and setting an anchor in the center of the number line (at "5") does not facilitate their correct estimation of the positions "4" and "6." Finally, all children showed also greater EE for the task using non-symbolic number format, especially at the right end of the number line, which may be interpreted as the manifestation of a position EE but also incorrectness in dot counting. Finally, there is an additional applicable value of obtained results in the context of the diagnosis of dyscalculia risk and the assessment of basic numerical abilities in preliminary school children. Two NLE tasks used in the present study are included in the computerized tasks battery (Gut, Matulewski & Goraczewski, 2016), which became a tool that aids the process of dyscalculia screening as well as the basic numerical abilities assessment in Poland. The results obtained in the present study strongly confirmed the application value of such computerized NLE tasks for examination of basic mathematical abilities in children.

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