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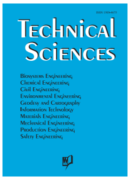

THE ANALYSIS OF THE RELATIONS BETWEEN POROSITY AND TORTUOSITY IN GRANULAR BEDS

Wojciech Sobieski, Seweryn Lipiński

Department of Mechanics and Basics of Machine Construction
University of Warmia and Mazury in Olsztyn

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Abstract

In the paper, functions describing different porosity-tortuosity relations were collected, and then the tortuosity values were calculated for a one granular bed consisting of spherical particles with normal distribution of diameters. Information about the bed porosity and particle sizes was obtained from measurements conducted for an artificial granular bed, consisting of glass marbles. The results of calculations were compared with the results of two other methods of tortuosity determination, performed for the same case (details are not described in this paper): the first of them uses the Path Tracking Method, the second one – information about the velocity components in a creeping flow (the Lattice-Boltzmann Method was applied to obtain the velocity field in the flow). The main aim of our article was to test whether the functions linking tortuosity with porosity, which are available in the literature, give similar results as the methods described above. To achieve this aim, the relative errors between results of calculations for the collected formulas and values from the both previous mentioned methods were calculated.

Introduction

Tortuosity is one of the most important parameters describing the porous beds. Tortuosity τ [m/m] is defined as the ratio of the actual path length inside pore channels L_p [m] to the thickness of the porous medium L_0 [m] (Bear 1972, Dias et al. 2006).

$$\tau = \frac{L_p}{L_0} \tag{1}$$

Correspondence: Wojciech Sobieski, Katedra Mechaniki i Podstaw Konstrukcji Maszyn, Uniwersytet Warmińsko-Mazurski, ul. M. Oczapowskiego 11, 10-957 Olsztyn, phone: 89 523 32 40, e-mail: wojciech.sobieski@uwm.edu.pl.

The tortuosity term became widespread among others by Kozeny (KOZENY 1927, SOBIESKI 2014), who corrected the value of the hydraulic drop occurring during fluid flow through a porous body by using this parameter.

A popular relationship between tortuosity and velocity, was presented by Carman as a correction of the Kozeny formula (CARMAN 1937, SOBIESKI 2014). In a general case, the path length L_p in the formula (1) may be understood as a geometrical quantity (geometric tortuosity) or as a flow property (hydraulic tortuosity) – see Figure 1.

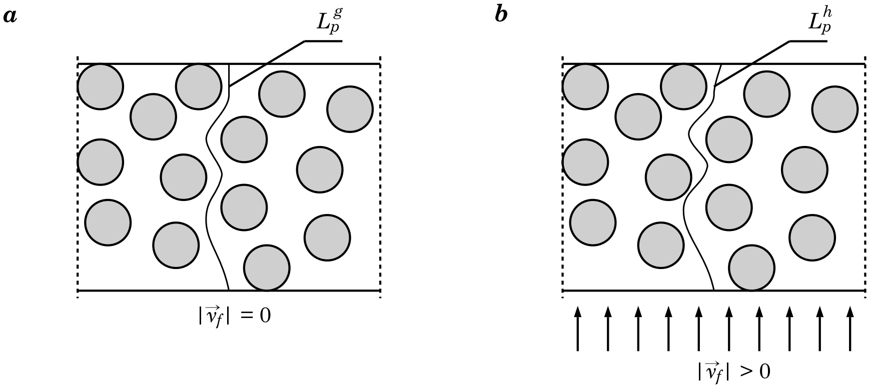

Fig. 1. The visualization of the tortuosity definition: *a* – geometric, *b* – hydraulic

The geometric tortuosity may be obtained in three different ways. The most common method is the use of a function linking it with other parameters characterizing geometry of the porous body. In this approach, it is usually assumed that the geometric tortuosity is a direct function of the porosity. In the literature, many functions may be found (derived empirically or analytically), where relationships between these quantities are proposed (YU, LI 2004, VALLABH 2009, AHMADI et al. 2014, ALLAN, SUN 2014, KONG et al. 2015). The second method involves the use of an experiment. Here, the computed tomography and image analysis (CT\IA) (WU et al. 2006, GOMMES et al. 2009, EBNER et al. 2013), acoustic methods (JOHNSON et al. 1982, KOCHAŃSKI et al. 2000, LI et al. 2010), optical methods (NWAIZU, ZHANG 2012), as well as the other methods (GAO et al. 2012) are used. In the third approach, the so-called Path Tracking Method (PTM) is used. In this method, the direction or shape of the porous space is tracked with the help of appropriate numerical algorithms (NAKASHIMA, KAMIYA 2007, STARLY et al. 2007, SOBIESKI et al. 2009). To reach it, firstly the geometry of the pore part is determined, and then its arrangement in the space is traced. When the path length L_p is known, the tortuosity may be finally calculated (SOBIESKI 2009, SOBIESKI, LIPIŃSKI 2013, SOBIESKI et al. 2016a, b).

The hydraulic tortuosity can be computed from fluid velocity fields. These fields may be obtained by means of the Computational Fluid Dynamics (CFD) method. The Lattice-Boltzmann Method (LBM) is the most popular (Feng et al. 2007, Nabovati, Sousa 2007, Matyka et al. 2008, Wang 2014). Another possibility is the application of the Finite Volume Method (FVM) or the Immersed Boundary Method (IBM).

Our article is a direct continuation of the studies described in a monograph of Sobieski et al. (2016). In these studies, different experimental, analytical, as well as numerical tests (e.g. the Lattice Boltzmann Method and the Immersed Boundary Method) related to the same porous bed, consisting of glass marbles, were performed.

Review of the functions linking porosity with tortuosity

Table 1 shows a review of the functions which describe the porosity and tortuosity relation. Two main groups of correlation functions may be distinguished. The first group is intended for systems containing squares (in 2D

Table 1

Review of formulas for calculating the tortuosity

Source	Application	Formula
1	2	3
Maxwell (1881) [Allan, Sun (2014)]	array of spheres in 3D, dilute suspension	$\tau = 1 + \frac{1}{2}(1 - e)$
Rayleigh (1892), [Allan, Sun (2014)]	array of cylinders in 2D	$\tau = 2 - e$
Bartell, Osterhof (1928) [Lanfrey et al. (2010), Ahmadi et al. (2014)]	packed beds	$\tau = 0.5\,\pi$
Carman (1937) [Lanfrey et al. (2010), Ahmadi et al. (2014)]	packed beds	$\tau = \sqrt{2}$
Mackie, Meares (1955) [Allan, Sun (2014)	diffusion of electrolytes in membrane	$\tau = \left(\frac{2 - e}{e}\right)^2$
Weissberg (1963) [Ahmadi et al. (2014), Allan, Sun (2014)]	bed of uniform spheres (applicable to overlapping, non-uniform spheres)	$\tau = 1 - 0.49 \ln e$
Bear (1972) [Dias et al. (2006, Ahmadi et al. (2014), Allan, Sun (2014)]	granular beds	$\tau = \frac{1}{e^C}$, where C is a constant
Kim et al (1987) [Allan, Sun (2014)]	isotropic systems, $0 < e < 0.5$	$\tau = e^{-0.4}$

cont. Table 1

1	2	3
Du Plessis, Masliyah (1988), [Ahmadi et al. (2014), Allan, Sun (2014)]	isotropic granular media	$\tau = \dfrac{e}{1 - (1 - e)^{2/3}}$
Comiti, Renaud (1989) [Tang et al. (2012), Ahmadi et al. (2014), Allan, Sun (2014)]	beds packed with spherical and cubic particles,	$\tau = 1 - C \ln e$, where C is a constant (0.63 in Tang et al. (2012) for cubic particles, 0.41 in Lanfrey et al. (2010) for packed beds)
Iversen, Jørgensen (1993) [Ahmadi et al. (2014), Allan, Sun (2014)]	sandy marine sediments, $0.4 < e < 0.9$	$\tau = \sqrt{1 + 2(1 - e)}$
Boudreau (1996) [Lanfrey et al. (2010), Ahmadi et al. (2014), Allan, Sun (2014)]	packed beds	$\tau = \sqrt{1 - \ln(e^2)}$
Koponen et al. (1996) [Tang et al. (2012), Ahmadi et al. (2014), Allan, Sun (2014)]	2D random overlapping mono-sized squares, $0.5 < e < 1$	$\tau = 1 + 0.8(1 - e)$
Koponen et al. (1997) [Tang et al. (2012), Allan, Sun (2014)]	2D random overlapping mono-sized squares, $0.4 < e < 0.9$	$\tau = 1 + 0.65 \dfrac{1 - e}{(e - 0.33)^{0.19}}$
Yu, Li (2004) Yu, Li (2004), Tang et al. (2012)]	2D square particles	$\tau = \dfrac{1}{2}\left[1 + \dfrac{\alpha}{2} + \alpha \dfrac{\sqrt{\left(\dfrac{1}{\alpha} - 1\right)^2 + \dfrac{1}{4}}}{1 - \alpha}\right]$ where $\alpha = \sqrt{1 - e}$
Matyka et al. (2008) [Kong et al. (2015)]	2D random overlapping mono-sized squares	$\tau = 1 - 0.77 \ln(e)$
Lanfrey et al. (2010) [Lanfrey et al. (2010), Allan, Sun (2014)]	bed of spheres	$\tau = 1.23 \dfrac{(1 - e)^{4/3}}{e\phi^2}$ where ϕ is a shape factor
Duda et al. (2011) [Allan, Sun (2014)	2D freely overlapping squares	$\tau = 1 + (1 - e)^{1/2}$
Pisani (2011) [Tang et al. (2012), Allan, Sun (2014)]	random, partial overlapping shapes	$\tau = \dfrac{1}{1 - \phi(1 - e)}$, where ϕ is a shape factor (0.73 in Tang et al. (2012) for cubic particles)
Tang et al. (2012) [Tang et al. (2012)	cubic particles	$\tau = \dfrac{3}{4} + \dfrac{1}{8}\sqrt{1 + \dfrac{1}{4}\dfrac{1 - e}{2 - e - 2\alpha}} + \dfrac{1}{8}\sqrt{1 + \dfrac{1 - e}{2 - e - 2\alpha}} + \dfrac{1}{4}\sqrt{1 - e}$
Liu, Kitanidis (2013) [Allan, Sun (2014)]	isotropic grain (spherical), staggered, $0.25 < e < 0.5$	$\tau = e^{0.28} + 0.15$

space), the second group applies to different granular beds (in 3D space). Formulas that may concern other cases are very rare. It is worth noting that besides the porosity, in some formulas, other quantities appear: a model constant or a factor related to the particle shape. In some cases, one equation may be used in both groups.

Determining the porosity of an artificial porous bed

Glass marbles (shown in Fig. 2*a*) were used in our investigations. An example of a granular bed consisting of such input material is shown in Figure 2*b*. The porosity of the bed was measured by using two graduated measuring cylinders with the volume of 250 ml. The first cylinder contained a bed sample; the second was filled with the distilled water. During the experiment, the water was slowly poured into the bed sample and the volume of pores was measured. The measurement was repeated 15 times, and in each case, the first cylinder was filled with new dry marbles. The obtained average porosity of the bed (a fraction of the volume of voids over the total volume) turned out to be equal to 0.41 ± 0.008 [-]. Obtained value is typical for loose random packing beds, which is 0.40-0.41 (RIBEIRO et al. 2010). It is worth mentioning that the obtained value is very close to the theoretical porosity (which is equal to 0.3954) of an orthorhombic system consisting of uniform spheres (COOKE, ROWE 1999).

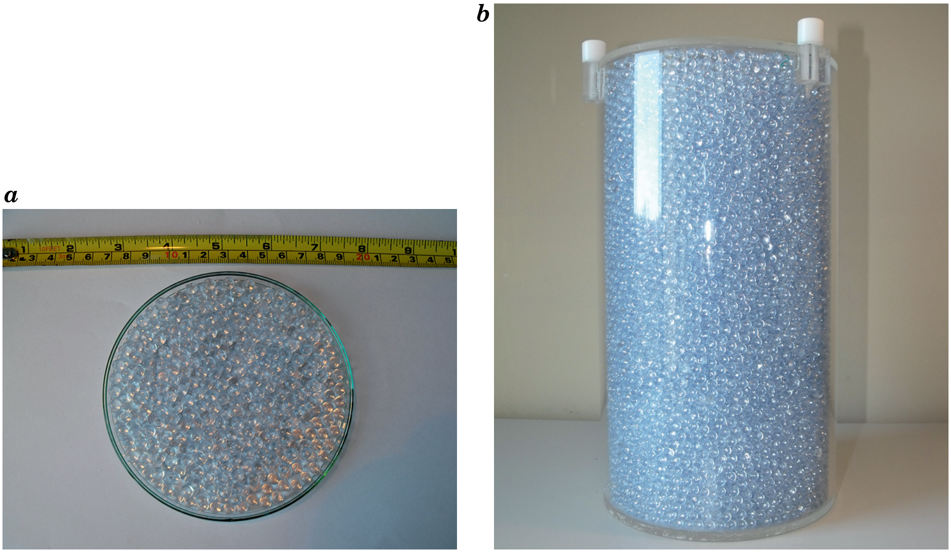

Fig. 2. The sample of glass marbles (*a*) and the exemplary granular bed (*b*)

The difference is caused by the fact that diameters in the artificial bed are not equal and the arrangement of the particles in the space is not regular.

In the next stage, 100 marbles were randomly chosen. The diameter of each marble in two random directions (perpendicular to each other) was measured with a micrometre screw with an accuracy of 0.01 mm. The average diameter of marbles was equal to 6.072 mm, with the standard deviation of 0.051 mm. The distribution of particle diameter is shown in Figure 3.

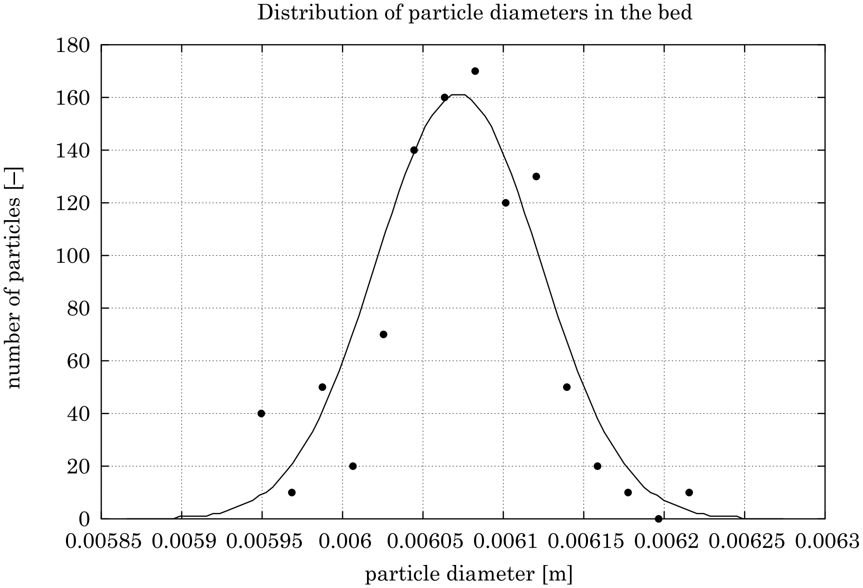

Fig. 3. The distribution of the particle diameter in the granular bed

Comparison

In the next stage of investigations, the tortuosity values for formulas collected in Table 1 and for porosity obtained in the experiment were calculated. Results of calculations are shown in Table 2. This table summarizes all results for comparison aims, although not all formulas are intended for granular beds. We can find some formulas give incorrect values. The formula (5) gives too high value of tortuosity while the formula (21) gives a non-physical result.

As it was mentioned above, the same porous bed as in our experiment, was earlier used for calculation of the geometric tortuosity with the Path Tracking Method (Sobieski 2009, Sobieski et al. 2012, *Pathfinder Project* 2013, Sobieski, Lipiński 2013, Dudda, Sobieski 2014, Sobieski et al. 2016a,

Table 2

Results of calculations

No	Formula	Tortuosity	Application in the example
1	MAXWELL (1881)	1.2950	weak
2	RAYLEIGH (1892)	1.5900	not applicable
3	BARTELL, OSTERHOF (1928)	**1.5708**	**strong**
4	CARMAN (1937)	**1.4142**	**strong**
5	MACKIE, MEARES (1955)	15.0393	not applicable
6	WEISSBERG (1963)	**1.4369**	**strong**
7	BEAR (1972)	–	–
8	KIM et al (1987)	1.4285	unknown
9	DU PLESSIS, MASLIYAH (1988)	**1.3826**	**strong**
10	COMITI, RENAUD (1989)	**1.3656 for C = 0.41**	**strong**
11	IVERSEN, JØRGENSEN (1993)	1.4765	weak
12	BOUDREAU (1996)	**1.6683**	**strong**
13	KOPONEN et al. (1996)	1.4720	not applicable
14	KOPONEN et al. (1997)	1.6197	not applicable
15	YU, LI (2004)	1.6594	not applicable
16	MATYKA et al. (2008)	1.6865	not applicable
17	LANFREY et al. (2010)	**1.4845 for** $\phi = 1$	strong
18	DUDA et al (2011)	1.7681	not applicable
19	PISANI (2011)	1.4184 for $\phi = 1$	unknown
20	TANG et al. (2012)	1.2586 for $C = 0.5$	not applicable
21	LIU, KITANIDIS (2013)	**0.9291**	**strong**

Table 3

Relative errors

Formula number	τ	τ^g	τ^h	δ^g [%]	δ^g [%]
3	1.5708	1.205	1.24	30.36	26.68
4	1.4142	1.205	1.24	17.36	14.05
6	1.4369	1.205	1.24	19.24	15.88
9	1.3826	1.205	1.24	14.74	11.50
10	1.3656	1.205	1.24	13.33	10.13
12	1.6683	1.205	1.24	38.45	34.54
17	1.4845	1.205	1.24	23.20	19.72

b) as well as for calculation of the hydraulic tortuosity by application of the Lattice-Boltzmann Method. In the first method, the tortuosity value was equal to 1.205, whereas in the second method to 1.24. Both values are smaller than all values presented in the Table 2 (besides the 21). The relative errors δ are shown in the Table 3. Geometric tortuosity τ^g and hydraulic tortuosity τ^h obtained in the former investigations were used as the reference values.

It can be assumed that the difference between the results obtained from the collected set of formulas and results of previous investigations may be caused by deviation of the particle diameters. The example presented above shows that the particles have normal distribution, but the variance is comparatively small. In many artificial beds, the diversity of particle sizes is much larger. Therefore, the formulas collected in Table 1 (obtained empirically in many cases) give higher values of tortuosity. In our opinion, this is due to the reasons described below.

When particles have different diameters, the shape of the pore space between them is more complicated. In consequence, sometimes the path may find a shorter way, but sometimes in turn it must omit the larger particles (as shown in Fig. 4). There is no doubt that it leads to a greater deviation of tortuosity values, but it is difficult to state at the current stage, which mechanism is the dominant one and so, whether the increase of the deviation of particles diameters is associated with an increase or a decrease in the average value of tortuosity in the bed. The problem of influence of the particle distribution on the path length is open and further studies are needed in this field.

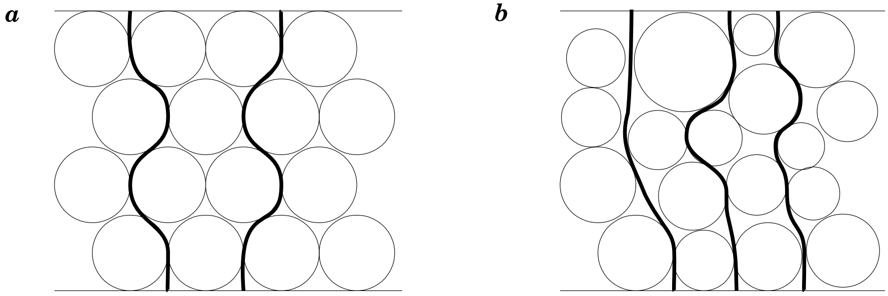

Fig. 4. Influence of the particle distribution on the path length for constant diameters (*a*) and diverse diameters (*b*)

Taking into account the fact that particle sizes in an artificial bed may be different, the porosity-tortuosity relations should have a more general form

$$\tau = f(e) \cdot f(\mu,\sigma^2) \tag{2}$$

or

$$\tau = f(e,\mu,\sigma^2) \tag{3}$$

where:

$f(e)$ – a function linking porosity and tortuosity for a bed consisting of particles with constant diameter,

$f(\mu,\sigma^2)$ – a correction function dependent on the average value and on its variance.

Summary and conclusions

The following conclusions can be formulated based on the above-discussed topics:

– Different mathematical formulas for determining the tortuosity may be found in the literature, but they give different results for the same data. Only for formulas designed for granular beds, the tortuosity value is within the range from 1.3654 to 1.6683. The relative error between these results is almost 20%, what is quite significant.

– In practice, it is impossible to indicate which equation in the Table 1 should be used in a specific case. Finding the next formulas does not solve this problem.

– Differences between tortuosity values obtained earlier (using the Path Tracking Method and the Lattice Boltzmann Method) and porosity-tortuosity functions available in the literature occur probably because the particle diameters in a artificial bed are not constant but they have a specific distribution.

– It can be assumed that tortuosity is probably higher in beds with higher deviation, than in beds where the particle diameters are constant.

– If the particle sizes in a granular bed are different, the porosity-tortuosity correlation functions should be corrected, for example by using a function dependent on the mean value and its variance. In this field, next investigations are needed.

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