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NUMERICAL EQUILIBRIUM ANALYSIS OF A STACK OF STEEL POST PALLETS

Józef Pelc

Department of Mechanical Engineering and Fundamentals of Machine Design
University of Warmia and Mazury in Olsztyn

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Abstract

A method for analyzing the equilibrium of a stack of loaded post pallets is presented. The finite element method was used to investigate the behavior of the bottom pallet in the stack during the addition of successive pallets. The stack was regarded as a self-stable multi-storey structure without bracings which is subjected to the weight of loaded pallets, horizontal forces resulting from sway and bow imperfections, and the impact of a forklift truck. The definite quadratic form of the tangent stiffness matrix after every increment in load was determined by nonlinear analysis to indicate the loss of post stability. An analysis of the stacking process of the evaluated pallets did not reveal a buckling trend in the posts of the bottommost pallet and demonstrated that the loss of equilibrium can lead to the collapse of the entire stack when a critical number of pallets is reached.

Introduction

Different types of pallets are used in warehouses for storing various products. Pneumatic tires are usually stored on steel post pallets. These pallets have a rigid bottom grid for storing tires. Posts are welded to the bottom grid, and the structural elements in the upper part of each post support the stacking of subsequent pallets. Pallets are stacked on top of each other to maximize storage space. However, the stack can become unstable when it reaches

Correspondence: Józef Pelc, Katedra Mechaniki i Podstaw Konstrukcji Maszyn, Wydział Nauk Technicznych, Uniwersytet Warmińsko-Mazurski, ul. M. Oczapowskiego 11, 10-719 Olsztyn, phone: +48 89 523 49 31, e-mail: joseph@uwm.edu.pl

a critical height, which can be caused by the buckling of the bottommost pallet or the collapse of the entire stack around the edge of its base. In daily practice, the determination of the safe maximum number of pallets in a stack poses a significant challenge for engineering staff. The above can be attributed to the absence of the applicable standards. Two pallet standards were previously in force in Poland (PN-M-78207: 1981, PN-M-78205: 1988). The first standard was revoked in 2012, and the second was revoked in 2015 without any replacements. These types of pallets have been rarely discussed in scientific and technical literature. WOLNY et al. (2014) investigated the stability and resistance of a box pallet to bending, stacking, free fall impact, lifting with a forklift truck, and horizontal impact with both edges and legs. They conducted analyses with the Finite Element Method (FEM) in the Femap/NEi Nastran system with the use of beam elements that are particularly useful in pallet design. Most published studies focus on pallet racks and on pallets for storing products in racks (on rack shelves) (BERNUZZI et al. 2015a, 2015b, SHAH et al. 2016). Unlike pallets, racks have post footings attached to the warehouse floor, and adjacent racks and pallet rack series are often connected by bracings, whereas post pallets are placed on the warehouse floor.

According to PELC (2017), a stack of post pallets should be regarded as a self-stable multi-storey structure without bracings. The above approach supports analyses of pallet stack stability with the use of the methods detailed in the standard applicable to steel structures (EN 1993-1-1:2005). The cited study proposes a calculation procedure and an exemplary analytical procedure for verifying the safety and stability of a stack of steel post pallets loaded with pneumatic tires.

This study relies on the FEM to propose a numerical method for determining the maximum number of loaded post pallets in a stack. The stability of structural components in the most loaded pallet, i.e. the bottommost pallet in the stack, and the stability of the entire stack were investigated by simulating the process of adding subsequent pallets to the stack. The vertical load of the bottommost pallet and the horizontal forces resulting from sway and bow imperfections increased with every additional pallet. The horizontal force generated by the forklift truck on the top pallet in the stack was taken into account based on the value calculated by TILBURGS (2001). The stack equilibrium was analyzed using the nonlinear incremental-iterative method, and the presence of a positive-definite or negative-definite quadratic form of the tangent stiffness matrix was determined after each increment. The absence of a positive-definite quadratic form was indicative of stack collapse. The FEM MSC.Marc/Mentat system was used in numerical simulations. In the analyzed case, pallet posts were stiff enough to prevent buckling, whereas a loss of static equilibrium was previously observed when the critical number of pallets was exceeded in the stack.

Computational Model

The pallet which was used to build the analyzed stack is shown in Figure 1. It was assumed that the rigid grid at the bottom the pallet was non-deformable.

The pallet computational model with specific dimensions is presented in Figure 2. The main structural elements of the pallet were: *1* – post (tubular section, 50×50×3), *2* – bed for the top pallet (angle bar, 50×50×4), *3* – crossbar (tubular section, 50×50×3), *4* – boom (flat, 50×8), *5* – bracket (tube, 50×3). In the analyzed pallet, dimensions a, b, c, d, h and e were determined at [m]: 1.25, 1.20, 1.07, 0.93, 1.50 and 0.09, respectively.

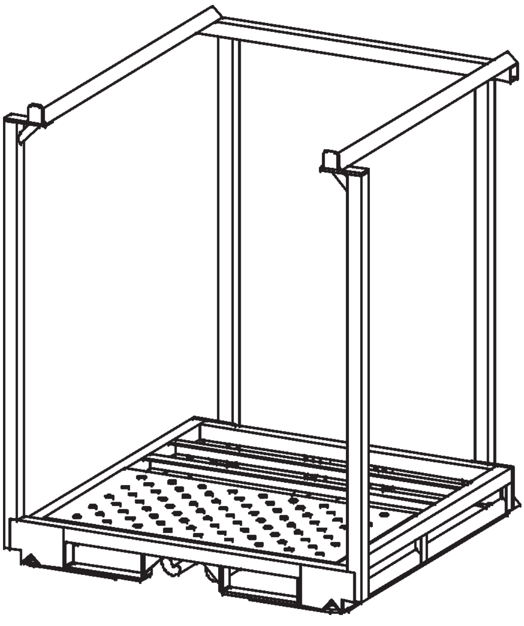

Fig. 1. View of the post pallet for storing pneumatic tires

The top pallet in the stack transfers load to the bottom pallet as a continuous load acting on the bed. In beds made of angle bars, the bar is bent around the axis of the minimum moment of inertia of the beam cross-section; therefore, it can be assumed that load will be transferred to the most rigid zones in the bed, i.e. points *A*-*D* which are supported by the posts. The rigid grid of the top pallet limits the relative displacement of points *A* and *B*, which is why they were joined by a non-deformable and weightless truss rod.

The bottommost pallet will be hereinafter referred to as the bottom pallet. The bottom pallet is subjected to the greatest load, and it determines the stability of the entire stack. The loads acting on successive pallets in the stack were reduced to points *A*-*D* of the bottom pallet. It was assumed that the lower ends of the posts are fixed in the pallet's rigid grid and that the bottom pallet touches the floor at four points. Friction forces prevent horizontal displacement of the pallet. In order to apply boundary conditions, all three possible rotations of bottom post nodes were blocked, and non-deformable elements were used to connect pallet support points on the floor with bottom post nodes and with the post nodes located on the pallet grid (Fig. 2).

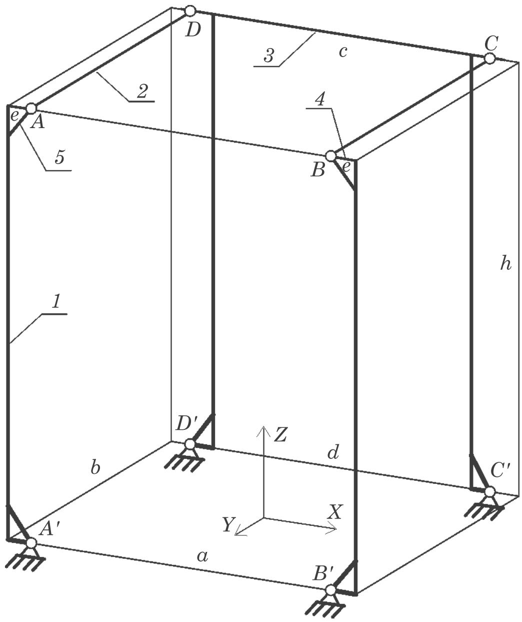

Fig. 2. Computational model of the post pallet (description in the text)

Sway and bow imperfections occurring in the system, whose values are specified in Standard EN 1993-1-1:2005, exert horizontal forces on the stack. The method of calculating these imperfections and the resulting values were presented in detail by PELC (2017). It should be noted that all possible translational and torsional sways were considered based on the recommendations formulated in the above Standard. The following inclinations were examined in this study: *DACB* (forward), *BADC* (torsional) and *BACD* (left). The acronym *DACB* indicates that points *D* and *A* move towards vector *DA*, whereas the remaining two points move towards vector *CB*. In the first two analytical cases, load-carrying capacity conditions were least satisfied by the bottom pallet in the stack (cf. PELC 2017). The forces acting on one point of the bottom pallet as a function of the number of pallets in the stack are presented graphically in Figure 3. The diagrams of increasing characteristic forces which were used in the analysis of stack displacement and static equilibrium are similar, but their values are smaller than the values of the calculated forces. Vertical forces acting as pairs of opposite forces (couples) originate from the horizontal forces acting above the bottom pallet and represent the moment of stack collapse (Fig. 4).

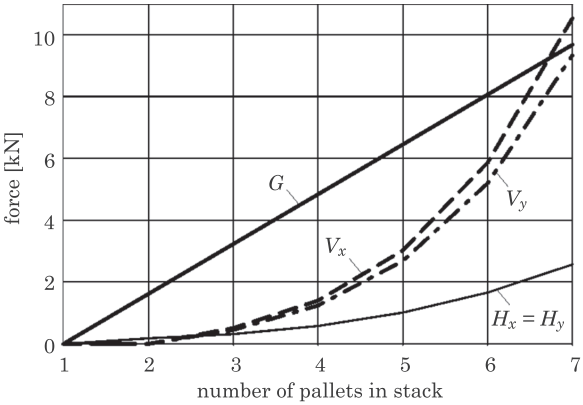

Fig. 3. Forces caused by weight (G) and imperfections (H – horizontal, V – vertical), acting on one point of the bottom pallet

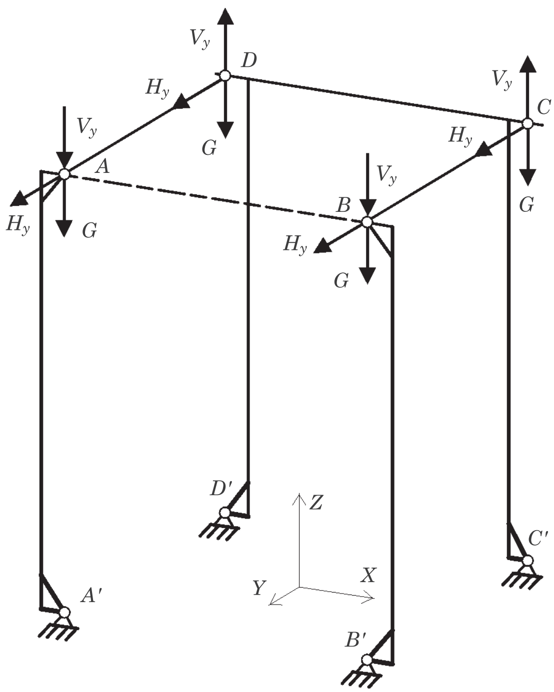

Fig. 4. Forces acting on the bottom pallet in case of *DACB* sway

A minor difference in the progression of vertical forces acting in planes parallel to planes *XZ* and *YZ* results from the difference in the distance between points *A* and *B* and points *B* and *C* ($AB < BC$), respectively.

Due to the significant values of horizontal forces acting on the stack (see Fig. 3) in addition to vertical forces, the maximum load-carrying capacity of the bottom pallet was calculated with a non-linear method. In the analyzed case, the distribution of forces is known, but the load, i.e. the number of pallets which cause stack instability, is unknown. In the total Lagrangian formulation, the increment in node displacement in the studied structure was determined from the following equation (BATHE 1982):

$$({}_0^t\mathbf{K}_L + {}_0^t\mathbf{K}_{NL})\,\Delta\mathbf{U}^{(i)} = ({}^{t+\Delta t}\beta)\,{}^{\Delta t}\mathbf{R} - {}^{t+\Delta t}_{\;\;\;\;\;0}\mathbf{F}^{(i-1)} \qquad (1)$$

where:

- ${}^{\Delta t}\mathbf{R}$ – vector of known loads in the first loading step,
- ${}^{t+\Delta t}\beta$ scale parameter which determines load in time $t+\Delta t$. The index in brackets is the iteration number,
- ${}_0^t\mathbf{K}_L$, ${}_0^t\mathbf{K}_{NL}$ – linear and non-linear (geometric) part of the stiffness matrix, respectively,
- ${}^{t+\Delta t}_{\;\;\;\;\;0}\mathbf{F}^{(i-1)}$ – nodal force vector resulting from node displacement.

When load reaches the value which causes system instability, small increments in load are accompanied by large increments in displacement, and the tangent stiffness matrix (the sum is given in brackets in equation 1) becomes singular. Furthermore, the iterative process ceases to converge. The assumption that the linear stiffness matrix ${}_0^t\mathbf{K}_L$ does not change significantly before system buckling and that the non-linear stiffness matrix ${}_0^t\mathbf{K}_{NL}$ is a multiple of its initial form leads to the so-called linear (initial) stability analysis of the eigenvalue problem (cf WOOD 1992)

$$({}_0^0\mathbf{K}_L + \lambda\, {}_0^{\Delta t}\mathbf{K}_{NL})\,\Delta\mathbf{U} = 0 \qquad (2)$$

The smallest eigenvalue λ_1 is determined to calculate critical load $\lambda_1{}^{\Delta t}\mathbf{R}$.

Two-noded beam elements with six degrees of freedom per node (three linear displacements and three angles of rotation) were used to analyze stack stability. The finite element model was composed of 294 elements (element 52 from MARC element library: straight, Euler-Bernoulli beam in space), and the assumed mesh density was validated with the mesh refinement method due to an approximation error. In a non-linear analysis examining the elastic behavior of the bottom pallet subjected to increasing load, the influence of large displacements (total Lagrangian formulation) was taken into account. The load imposed by additional pallets was increased in ten equal increments. The iterative process was conducted according to the Newton-Raphson procedure and was terminated when the displacement convergence criterion was satisfied.

Results and Discussion

A steel post pallet can be regarded as a frame whose legs are fixed in a non-deformable floor (grid at the bottom of the pallet). An initial/linearized stability analysis of the bottom pallet was performed according to (2). Vertical unit forces were applied to points *A*, *B*, *C* and *D*, and the lowest eigenvalue λ_1 was determined by solving the eigenvalue problem. The resulting critical load value was 62.7 kN. The fundamental mode of buckling was the lateral displacement and rotation of the upper part of the pallet (Fig. 5).

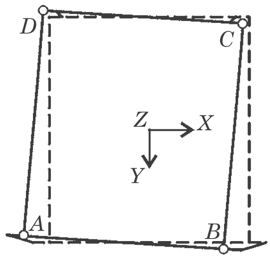

Fig. 5. The first mode of bottom pallet buckling

In sways *DACB*, *BADC* and *BACD*, vertical reactions were examined at points where the bottom pallet was supported by the warehouse floor (Fig. 6) and at points of displacement of the forces applied to the pallet, i.e. points *A*, *B*, *C* and *D* (Fig. 7*a*, *b*). The diagrams presenting the changes in the values of vertical reactions acting on the bottom pallet indicate that all reactions had positive values up to six pallets in the stack, but when the seventh pallet was added, the reactions of some supports reached zero; therefore, negative reactions should be applied to balance the stack. This approach is possible in the adopted

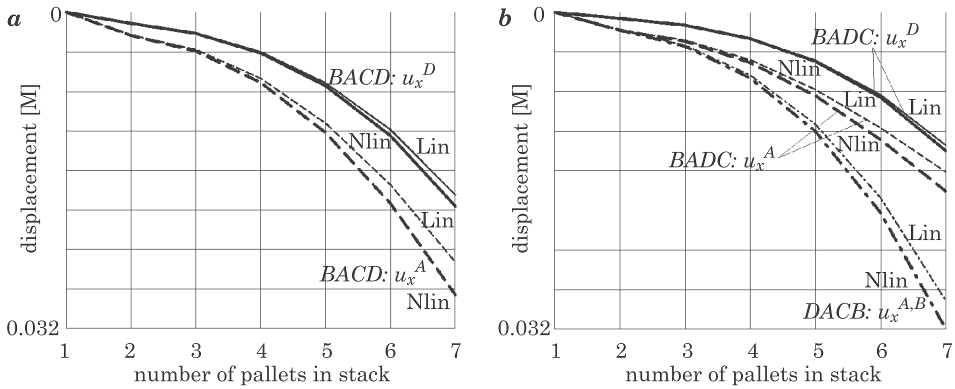

Fig. 6. Vertical reactions at points where they can assume negative value

model, but in reality, the pallet and the floor are bound by one-sided constraints, and the achievement of zero reaction force should be regarded as a loss of static balance. Changes in floor reaction forces acting on the bottom pallet in the three analyzed sways are presented in Figure 6, but only at points where a given number of pallets can change the sign of these reactions. These points are located opposite to the sway. For example when the stack sways to the left (*BACD*), these are points located on the right side of the pallet, i.e. *B*' and *C*'. Reaction forces increase monotonically in the remaining supports.

The displacement of points *A*-*D* on the bottom pallet increases monotonically with an increase in the number of pallets in the stack. In a stack with six pallets, the greatest displacement of 20.2 mm in the direction of the *y*-axis was noted in points *A* and *B* with *DACB* sway. Points *A* and *B* were least displaced in torsional sway *BADC*. The results of the linear analysis are presented in Figures 7*a* and 7*b*. In a stack with six pallets, the linear displacement of point *A* deviated most significantly from non-linear displacement in sway *BACD* (9.7%) and a similar deviation occurred in sway *BADC* (9.2%).

Displacement of points *A*, *B* and *D* on the bottom pallet as a function of the number of pallets in the stack in sways *DACB* and *BACD*.

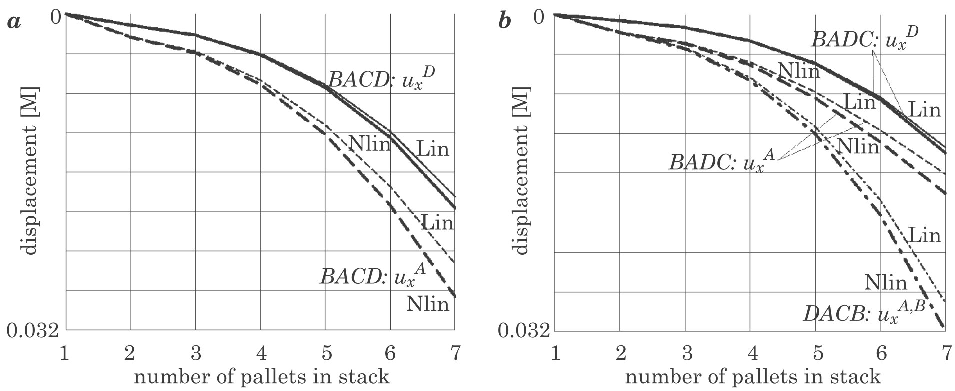

Fig. 7. Displacement of points *A* and *D* on the bottom pallet as a function of the number of pallets in the stack in sway *BACD* (*a*); displacement of points *A*, *B* and *D* on the bottom pallet as a function of the number of pallets in the stack in sways *DACB* and *BADC* (*b*): Lin – linear analysis, Nlin – non-linear analysis

It should be noted that the progression of displacement changed rapidly in an incremental manner when 5 pallets were stacked (refer to the variant of sway *DACB* in Figure 7*b*), which resulted from the rapid increase in horizontal forces mainly due to bow imperfection. The values of these forces depend on the values of the compressive normal forces acting on the posts, and they increase with the number of pallets in the stack.

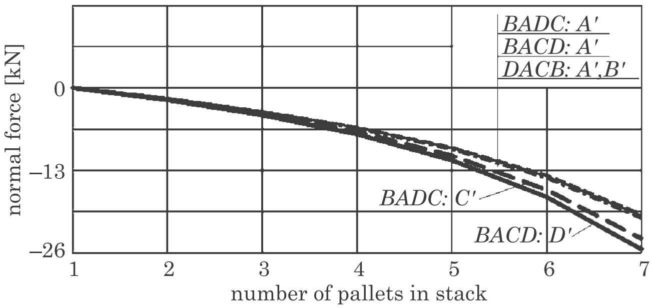

Fig. 8. Compressive normal forces acting on posts

Diagrams of compressive normal forces acting on pallet posts with different sways forms presented in Figure 8. In the case of the most dangerous sway *BACD*, the compressive force of 26 kN is far from the critical post force of 62.7 kN.

Conclusions

Vertical loads and equivalent horizontal loads acting on a stack as a result of sway and bow imperfections can be determined when a pallet stack is regarded as a multi-level self-stable structure.

The stability of stacked loaded post pallets can be effectively analyzed using the general non-linear incremental-iteration FEM procedure.

An analysis of the displacement history of the characteristic points on the bottom pallet indicates that geometric non-linearities exert a moderate influence on displacement. The greatest differences in displacement between linear and non-linear analysis were determined at 10%.

In an analysis of the equilibrium of a stack of post pallets loaded with pneumatic tires, static balance was lost when the seventh pallet was added. None of the posts in the bottom pallet buckled in the analyzed sways, which suggests that the evaluated post cross-sections confer high flexural stiffness.

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