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ANALYSIS OF SELECTED MATHEMATICAL MODELS OF HIGH-CYCLE S-N CHARACTERISTICS

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Abstract

The paper presents two approaches of determining S-N fatigue characteristics. The first is a commonly used and well-documented approach based on the least squares method and staircase method for limited fatigue life and fatigue limit, accordingly. The other approach employs the maximum likelihood method. The analysis of the parameters obtained through both approaches exhibited minor differences. The analysis was performed for four steel construction materials, i.e. C45+C, 45, SUS630 and AISI 1045. It should be noted that the quantity of samples required in the second approach is significantly smaller than with the first approach, which translates into lower duration and costs of tests.

Introduction

The designing of new structural elements subjected to stress that is variable in time, i.e. bicycle frames, load-bearing parts of ship hulls, bogies frames or bodies of rail vehicles requires determining their endurance or fatigue strength for the assumed life ERRI B12 RP 17 8 edition (1996), KOZAK and GÓRSKI (2011), PN-EN 14764 (2007). These calculations require obtaining

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the S-N fatigue characteristics, which, for most construction materials, is divided into two scopes. The first scope is related to limited fatigue strength, which is usually expressed with the following equation:

$$\log(N) = m \log(S) + b \quad (1)$$

where:

- N – number of cycles,
- S – stress amplitude [MPa],
- m – slope coefficient,
- b – intercept term.

The other scope is related to unlimited fatigue life, which, at its upper side, is limited by the fatigue limit. The description of this area will be presented in the further part of this article.

Fatigue calculations often include calculating the safety factor. In the case when fixed amplitude of stress on the structural element is assumed, the safety factor is calculated according to the following KOCAŃDA and SZALA (1997) correlations:

$$\delta = \frac{Z_N}{S} \quad (2)$$

where:

- δ – safety factor for constants stress amplitude,
- S – stress amplitude,
- Z_N – fatigue strength for the required fatigue life according the formula (1).

Normative characteristics as per PN-EN 1993-1-9, (2007) requirements or guidelines of classification associations, e.g. ERRI B12 RP 17 8 edition (1996), HOBACHER (2009), KOCAK et al. (2006) may be used for the calculations. Examples of diagrams for different categories of welded joints are presented on Figure 1. The individual characteristics refer to various categories of welded joints, referred to as FAT classes. The values assigned in the legend define the value of fatigue strength for strength $2 \cdot 10^6$.

Application of the analytical methods requires credible parameters for equation (1), which are estimated from experimental data. 2 models are used for estimating these values. The first model, marked as I, is the conventional approach defined in normative documents, e.g. ASTM E-739-91 (2006), ISO-12107 (2003), PN-EN-3987 (2010), PN-H-04325:1976 (1976), which the documents were compared in the paper STRZELECKI et al. (2015). The other model,

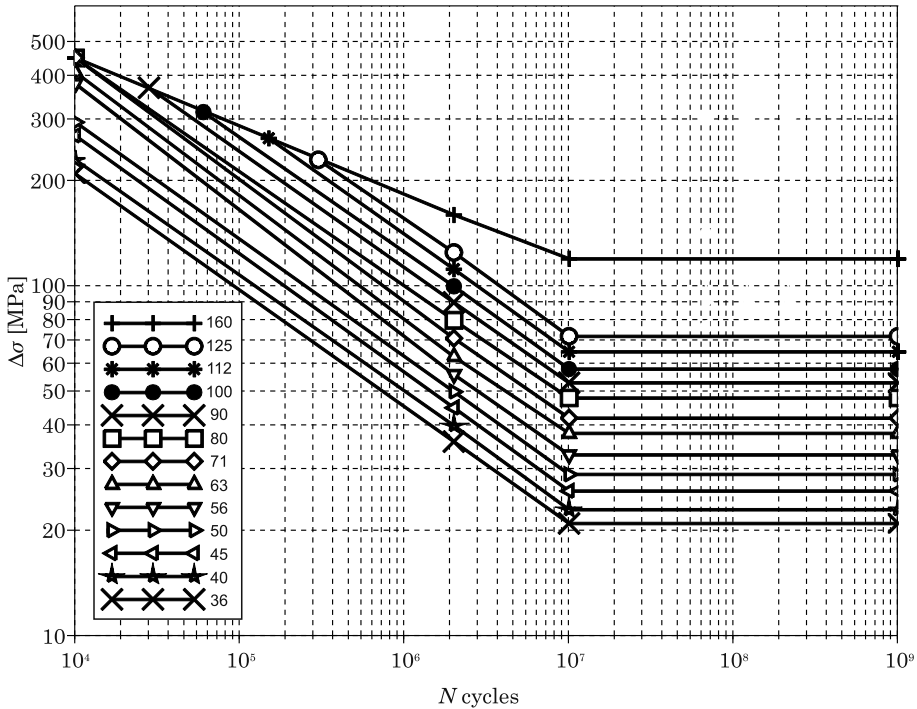


Fig. 1. Examples of graphs for different categories of fatigue of welded joints according Hobbacher Source: based on HOBACHER (2009).

marked as II, is an alternative approach, the description of which is contained in the paper PASCUAL and MEEKER (1999). Parameter estimate procedure can be found in the paper COVA and TOVO (2016). Comparisons of the model II, particularly focusing on the accuracy and sensitivity to changes in test parameters, have been a subject of numerous papers. Its further verification is justified, however.

The purpose of this paper is presentation of the method II used for determining the S-N characteristics in a manner more accurate than hitherto made in scientific literature. It was decided that the analysis of this method will be carried out based on own experimental results, as well as existing scientific resources, which should render the conclusions as objective as possible.

Authors didn't compare other models like STROHMEYER (1914), PALMGREN (1924), WEIBULL (1949), STÜSSI (1955), CASTILLO (1985), KOHOUT and VACHET (2001). This can be found in others papers like KUREK et al. (2014), KOHOUT and VECHET (2001), BANDARA et al. (2016) and books CASTILLO and FERNÁNDEZ-CANTELI (2009) or SZALA and LIGAJ (2011). In nonlinear models, coefficient

S_0 is referred as fatigue limit, but it is only mathematical coefficient. This value must be lower than the lowest stress level of the sample, and it is usually differs from the 50% fatigue limit, see paper GOGGIO and ROSSETTO (2004). For this reason only model PASCUAL and MEEKER (1999) was chosen.

Conventional method of determining the S-N characteristics

It should be noted that the diagrams presented above apply to 75% confidence, with 95% reliability (HOBACHER 2008, PN-EN 1993-1-9 2007). When using a new material or manufacturing technology, the fatigue characteristics of the given element is often not available; it is also commonly known that fatigue strength of this elements changes. Due to this, fatigue examinations, aiming at determining the S-N relationship are carried out. They are most often performed according to standardized guidelines (e.g. ASTM E-739-91 2006, ISO-12107 2003, PN-EN-3987 2010, PN-H-04325:1976 1976). The documents referred to above apply to determining a fatigue relationship in the scope of limited fatigue life. The number of tests to be performed is different, depending on the documents referred to. According to the Polish standard, the minimum number of samples tested is 15. It is recommended that the tests are performed at 5 stress levels, with 3 samples each. ISO-12107 (2003) standard, on the other hand, requires that at least 7 tests are made as part of preliminary examination, and that at least 28 sample are used for determining reliability. The ASTM E-739-91 (2006) standard was used for deciding on the number of levels; the standard specifies the replication requirements as follows:

$$PR = 100 \left[1 - \left(\frac{Sl}{n} \right) \right] [\%] \quad (3)$$

where:

Sl – number of stress level,

n – the total number of specimens.

The recommended replication percentage values are presented in Table 1.

PN-EN-3987, (2010) standard, on the other hand, does not specify the quantity of samples definitely. It merely contains a guideline saying that the tests should be started at a load at which a crack may be expected, with around 105 cycles. The tests should be carried out at least 5 stress levels.

Another approach can be found in the Guidelines of the International Institute of WELDING HOBACHER (2008), where it is suggested that the tests are carried out at 2 stress levels, for at least 10 samples. The tests should be performed within the range of fatigue strength for $10^5 \div 10^6$ cycles.

Guidelines recommend that the percent replication for various tests

Table 1

Type of Tests	Percent Replication
Preliminary and exploratory (research and development tests)	17 to 33
Research and development testing of components and specimens	33 to 50
Design allowable data	50 to 75
Reliability data	75 to 88

Source: ASTM E-739-91 (2006).

Researchers often limit the number of test samples due to extensive time requirements and high costs generated by the procedures. For instance, in order to perform 10^5 cycles at 30 Hz stress frequency, the total duration of the test must be around 55 min. To perform 10^6 cycles, the test would last 9 hours. Sample preparation time not included. It must be noted that the strength testing machines often allow to obtain a much lower frequency, e.g. 5 Hz, or smaller, which significantly increases the test duration.

The results obtained from the tests allow to determine the S-N characteristics. Linear regression according to correlation (1) is commonly applied. The least squares method, as specified in w ASTM E-739-91 (2006), HOBACHER (2008) among others, is used to determine parameters of equation (1).

An example of scheme for evaluation of the fatigue diagram is presented on Figure 2. Note that a fatigue limit, marked on the figure as Z_G often applies to construction materials, e.g. steel. The step method, described in the KOCAŃDA and SZALA (1997), LEE et al. (2005) or in the ISO-12107 (2003) standard is usually used to determine this value. The test must be performed on at least 15 samples to determine the fatigue limit using the staircase method. Assuming the value of basic quantity of cycles N_G as $5 \cdot 10^6$, the total duration, at an assumed stress frequency of 30 Hz, is ~ 470 hours. This is equivalent to 20 days.

The test duration calculations presented above show that the time required to determine the full stress characteristics is at least a month. This generates high time requirements and costs, which in turn causes significant limitations in performing such examinations. Due to this, the stress examinations are reduced to minimum by the broadest possible employment of characteristics already at hand. This can lead to obtaining inaccurate calculation results, however. The procedure described above will be referred to as model I.

Alternative method of determining the S-N characteristics

As exhibited in previous works of other authors, for instance STRZELECKI et al. (2016), STRZELECKI and SEMPRUCH (2016), an alternative method may be used. The advantage of that method is the ability to evaluate a full fatigue

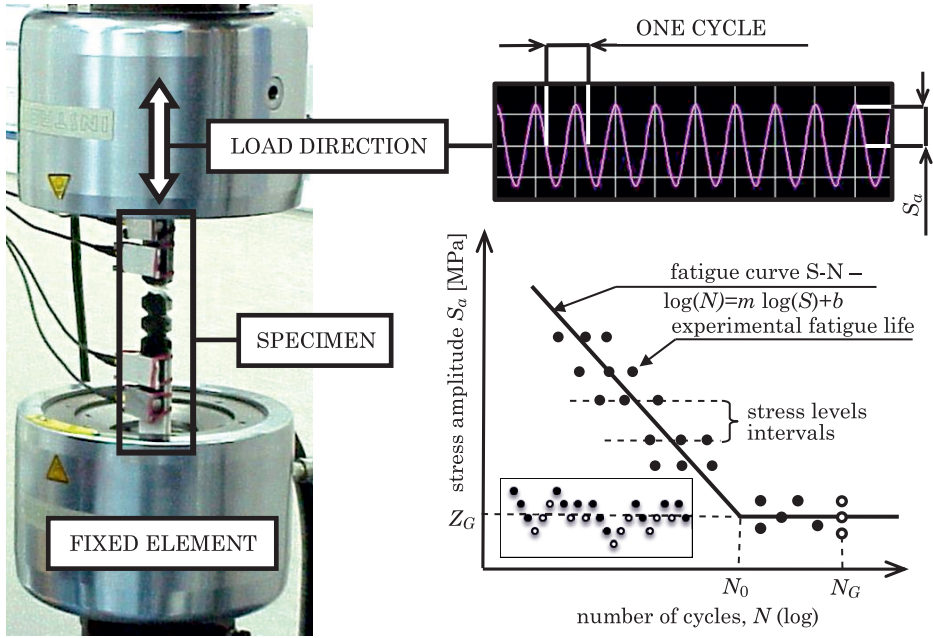


Fig. 2. Scheme of estimate fatigue curve in high cycle range and fatigue limit using standard test machine

relationship for a smaller experimental data set. This model assumes that the fatigue strength logarithm and the fatigue limit have a normal distribution, which the distributions can be expressed as follows:

$$f(N) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{(\log(N) - (m\log(S) + b))^2}{\sigma_n^2}\right) \tag{4}$$

where:

σ_n – standard deviation,

$$f(S) = \frac{1}{\sqrt{2\pi\sigma_s^2}} \exp\left(-\frac{(S - Z_G)^2}{\sigma_s^2}\right) \tag{5}$$

where:

σ_s – standard deviation.

This is considering that cracks may occur in case of applying stress higher than the Z_G threshold, and in case of obtaining strength higher or equal to the strength described in equation (5). Also, considering that these values are

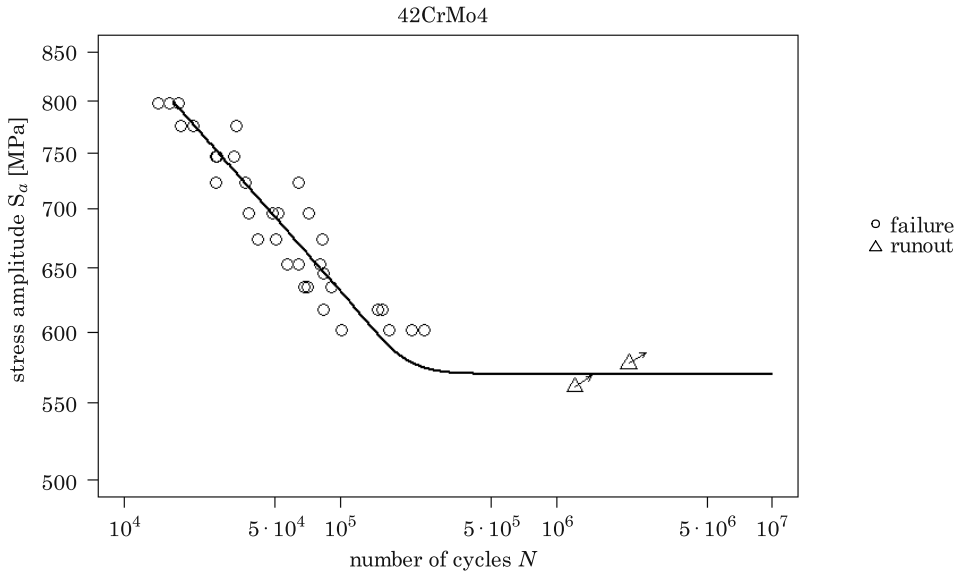


Fig. 3. Example of S-N curve for 42CrMo4 steel (own work) estimated by alternative method

random, it should be assumed that the likelihood of fracture will be equal to the likelihood of fulfilling function (5) and function (6). With this assumption, the probability of fracture for the limited and unlimited fatigue life scope may be expressed as follows (LORÉN, LUNDSTRÖM 2005):

$$P(N < N_i) = \Phi \left(\frac{\log N_i - (a \cdot \log S_i + b)}{\sigma_v} \right) \cdot \Phi \left(\frac{S_i - Z_s}{\sigma_v} \right) = q \quad (6)$$

where:

Φ – the normal distribution function,

q – the probability of specimen failure.

The maximum likelihood method was employed for determining the values of parameters of equation (7). The advantage of this statistical method is the possibility of taking into account the samples that did not fracture during the test. Reliability function for this method is expressed as follows (PASCUAL, MEEKER 1999):

$$L(\theta) = \varphi\left(\frac{\log N_i - (a \cdot \log S_i + b)}{\sigma_v}\right)^{\delta_i} \cdot \varphi\left(\frac{S_i - Z_{xe}}{\sigma_{xe}}\right)^{\delta_i} \cdot \left(1 - \left(\varphi\left(\frac{\log N_i - (a \cdot \log S_i + b)}{\sigma_v}\right)\right)^{\delta_i}\right) \cdot \varphi\left(\frac{S_i - Z_{xe}}{\sigma_{xe}}\right)^{1-\delta_i} \quad (7)$$

where:

φ – the normal distribution density function.

$$\delta_j = \begin{cases} 1 & \text{if } N_i \text{ if specimen failure} \\ 0 & \text{if } N_i \text{ if specimen runout} \end{cases} \quad (8)$$

The example of a diagram obtained using the method presented above is shown in Figure 3. The presented diagram applies to a likelihood of 50%.

Experimental method and results

Data derived from fatigue tests examinations for C45+C steel were used to compare the two models. The fatigue test was performed on a test stand described in the paper STRZELECKI and SEMPRUCH (2012). Test has been carrying out using the rotary-bending fatigue machine. The test stand had been verified earlier. The machine verification involved determining the maximum error of the bending moment applied. The calculations of that value were made compliant with the norm ISO-1143 (2010) and it was 1.15%. The admissible value here was 1.3%. The admissible value here was 1.3%. Specimen was made from bar with diameter 10 mm. In measurement places there was $\varnothing 5$ mm and radius 25 mm. It should be noted that the samples were made of a drawn rod in as-delivered condition. Static properties of the tested material are presented in Table 2.

Results of the experimental tests of material C45+C, along with characteristics determined according to model I and II are presented on Figure 4a. The tests were performed on 31 samples for limited fatigue life, and 19 for fatigue limit (7 cracked, and 12 reached the limit number of cycles N_G). It must be noted that the characteristics presented are estimated for a 50% likelihood of failure. All the calculations were made in R Core Team (2015) software. Additionally, Figure 4b presents the results obtained using the staircase method.

Table 2

Properties of materials C45+C from Strzelecki and Sempruch

Property of material	Value
Ultimate strength S_u [MPa]	826
Yield stress S_y [MPa]	647
Hardness HB	232

Source: STRZELECKI, SEMPRUCH (2012).

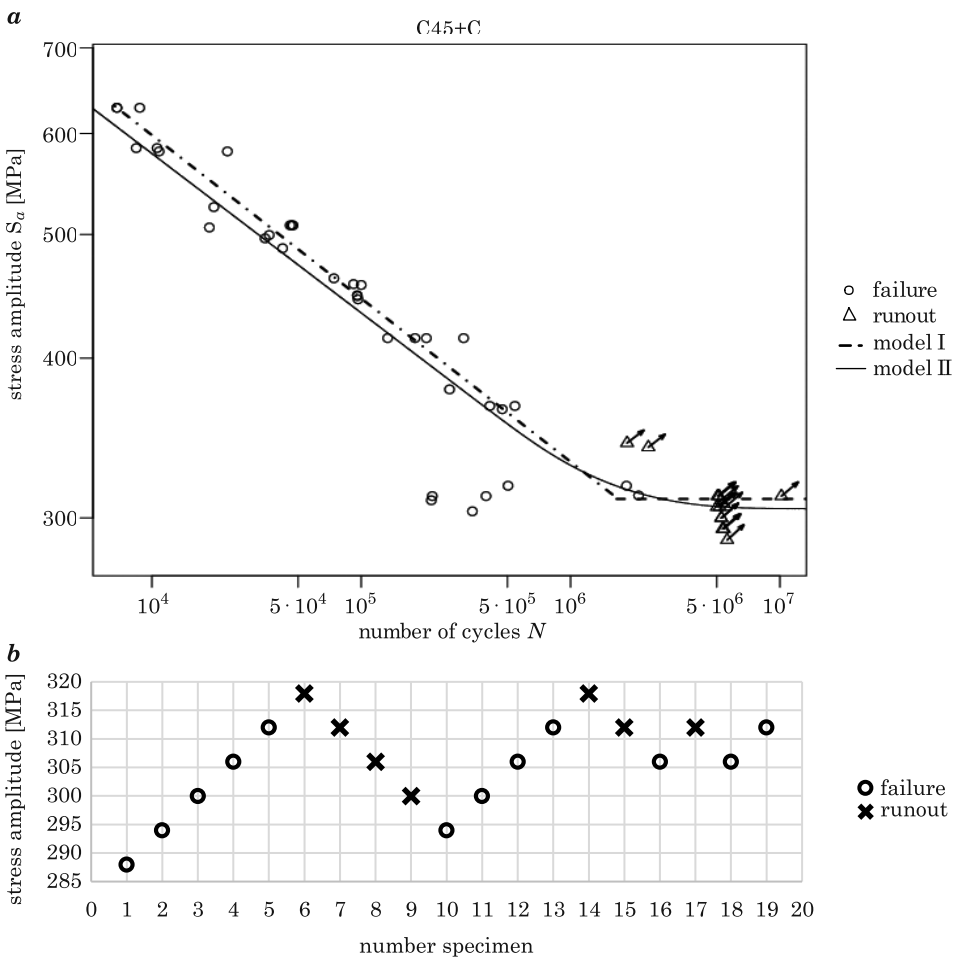


Fig. 4. S-N curve and fatigue data for steel C45+C: *a* – S-N curves, *b* – staircase method

Comparison of methods I and II

Comparison of the methods discussed was also carried out for steel 45, the test results for which are presented in the LING and PAN (1997). The experiment was performed on samples with a notch of stress concentration factor equal 2. The samples were loaded to axial load with asymmetry factor equal -1. The characteristics obtained are presented on Figure 5a.

Also compared were the results obtained for steel SUS630 (EN equivalent: X5CrNiCuNb16-4). The description of tests performed on this material can be found in the paper MOHD et al. (2015). The tests were performed on smooth samples, at stress frequency of 100 Hz. Load asymmetry factor was -1. The experiment was performed with axial load. The characteristics obtained are presented on Figure 5b.

The last material for comparison was AISI 1045 steel. The test results were derived from paper AVILES et al. (2013). The tests were performed on smooth samples, stressed from rotational bending at 33.3 Hz frequency. The characteristics obtained are presented on Figure 5c.

Results of estimated parameters for all the materials are shown in Table 3. No fatigue limit was determined for material 45, since the required number of experimental data was not available. Full characteristics was determined in this case.

Table 3

Estimated parameters for method I i II

Material	Method	a	b	σ_v	Z_s	σ_s
C45+C	I	-7.8	25.7	0.28	310.3	11.4
	II	-8.0	26.0	0.29	304.9	20.0
45	I	-8.5	26.5	0.12	–	–
	II	-8.6	26.9	0.12	301.8	30.0
SUS630	I	-9.2	31.9	0.17	651.5	6.7
	II	-9.9	34.1	0.27	649.6	7.0
AISI 1045	I	-8.1	26.5	0.13	352.0	7.7
	II	-8.6	28.0	0.20	351.0	7.2

Calculations for evaluating the value of fatigue strengths according to correlation (1) were performed in order to compare the estimated parameters according to method I and II. Then, the difference between the values obtained was calculated. It must be mentioned that the calculations were performed for different stress amplitude levels, which the amplitude corresponded to the scope of the high-cycle tests. The results obtained are presented in Table 4.

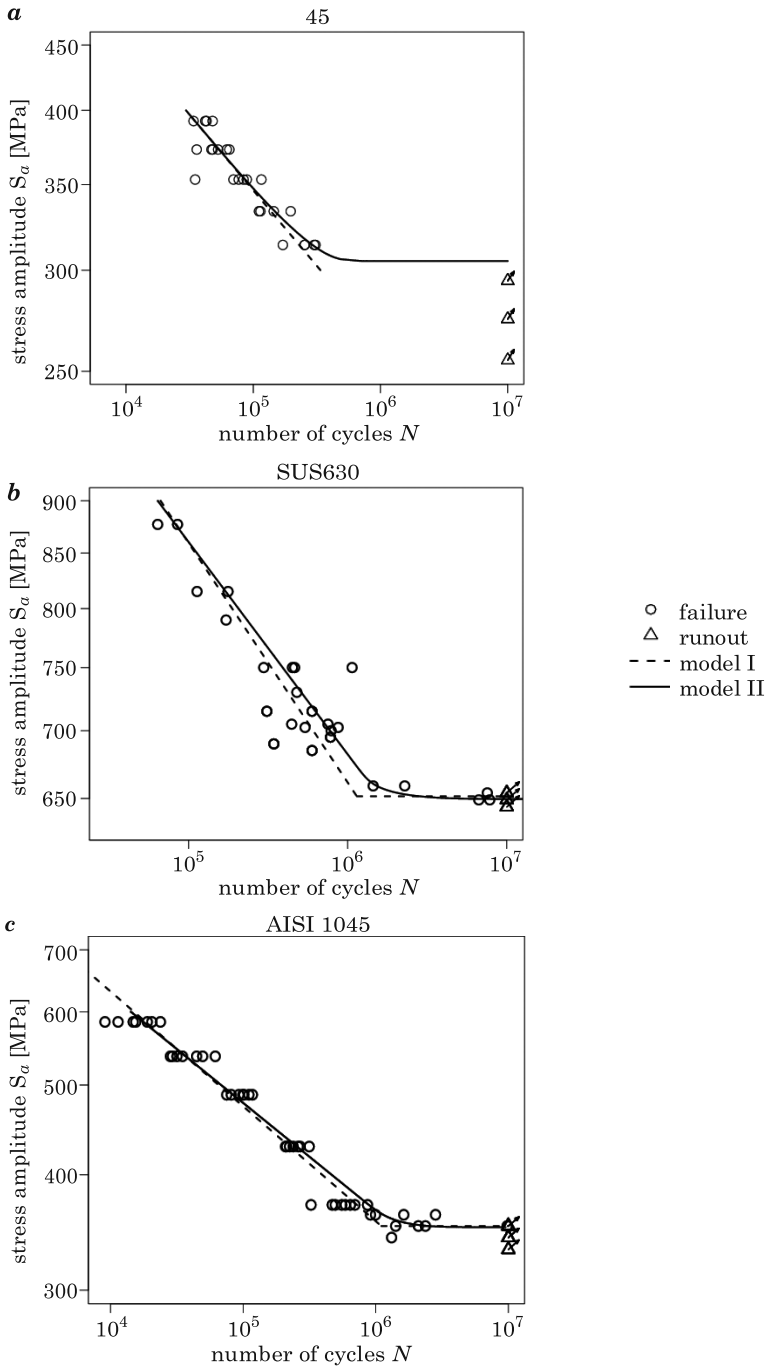


Fig. 5. S-N curve for: *a* – steel 45 (from paper LING and PAN 1997, *b* – steel SUS630 (from paper MOHD et al. 2015), *c* – steel AISI (from paper AVILES 2013)

Table 4

The values obtained by the sustainability model I i II

Material	$\log(S)$	$\log(N_1)$ – method I	$\log(N_2)$ – method II	$\log(N_1)$ – $\log(N_2)$	σ_s – method I
C45+C	2.80	3.84	3.81	0.02	0.28
	2.65	4.97	4.88	0.09	
	2.54	5.82	5.68	0.14	
45	2.60	4.47	4.46	0.01	0.12
	2.56	4.86	4.86	0.0	
	2.51	5.29	5.30	0.01	
SUS630	2.91	5.18	5.21	0.03	0.17
	2.88	5.49	5.56	0.07	
	2.84	5.84	5.95	0.11	
AISI 1045	2.77	4.25	4.27	0.02	0.13
	2.69	4.92	4.90	0.02	
	2.57	5.95	5.86	0.09	

Summary and conclusions

The characteristics obtained for material C45+C according to model I and II are different. They are shown on the graphical presentation in Figure 4a. Considering that the model II characteristics is shifted to the left, which results in underestimation of strength, the situation may be deemed safe (conservative). Moreover, standard deviations of the number of cycles for the limited and unlimited strength scope are higher in case of model II. This is related to a proportionally higher dispersion of the strength of samples around the fatigue limit.

For material 45, the differences between the estimated parameters according to the described methods in terms of limited fatigue life are significantly smaller than for the previous material. Standard deviations were equal. Determination of the fatigue limit for steel 45 was possible only by application of model II. This fact suggests superiority of the alternative method.

When analysing the characteristics of SUS630 steel, a clear difference in limited fatigue life was noted. According to the authors, this is related to the higher number of samples for stress level nearing the fatigue level, compared to high stress levels. This should be taken into consideration for any subsequent fatigue tests. When evaluating the fatigue limit, the differences should be assumed as marginal (difference in value smaller than standard deviation).

For material AISI 1045, the differences between the estimation of fatigue strength for limited fatigue life around the fatigue limit were the highest and noticeable (see Fig. 5c). The differences in the estimated fatigue limit were below 50% of standard deviation. They may be deemed as insignificant, therefore.

To conclude, the following conclusions has been formulated:

a) The method II described herein reduces the required number of samples for evaluating SN characteristics.

b) The time savings from the performance of the test is significant, considering that the method does not require a large amount of samples for unlimited fatigue life tests, where the test duration is the highest.

c) In contrast to the conventional method marked as I, it should be noted that the difference in values of estimated parameters is small.

d) By analysing the results presented in Table 3 and Table 4 it may be concluded that the differences in the fatigue limit and the estimated fatigue life are below standard deviations.

e) In the case of estimating the fatigue life, the largest differences may be noted around the stress nearing the fatigue limit.

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