Technical Sciences, 2017, 20(3), 259–273

Quarterly peer-reviewed scientific journal
ISSN 1505-4675
e-ISSN 2083-4527
TECHNICAL SCIENCES
Homepage: www.uwm.edu.pl/techsci/

APPROXIMATED MATHEMATICAL MODEL OF HYDRAULIC DRIVE OF CONTAINER UPTURNING DURING LOADING OF SOLID DOMESTIC WASTES INTO A DUSTCART

Oleg Berezyuk, Valeriy Savulyak

Faculty of Machine Building and Transport
Vinnytsia National Technical University
Vinnytsia, Ukraine

Received 2 November 2016, accepted 21 June 2017, available online 22 June 2017.

Key words: container upturning, Laplace transform, dustcart, hydraulic drive, mathematical model, solid domestic wastes.

Abstract

A simplified mathematical model of hydraulic drive of container upturning during loading of solid domestic wastes into a dustcart is proposed. Approximated analytical time dependencies of pressure in a delivery pipe of a hydrocylinder, angular velocity and the angle of container upturning are obtained. Approximated duration dependence of container upturning from main parameters of hydraulic drive is detected. Optimal values of main parameters of working organs, which ensure minimum time of container upturning during loading of solid domestic wastes into a dustcart are defined.

Introduction

In obedience to statistical data, the yearly volume of solid domestic wastes (SDW), produced in the populated areas of Ukraine, exceeds 46 million m^3. Their overwhelming majority is buried on polygons and dumps. SDW collecting is a main task of sanitary cleaning of the populated areas and is carried out by more than 4.1 thousand special cars (dustcarts), and that is why it is

Correspondence: Oleg Berezyuk, Faculty of Machine Building and Transport, Vinnytsia National Technical University, 95 Khmelnytske shose, Vinnytsia, Ukraine, 21021, e-mail: berezyukoleg@yandex.ru

associated with considerable financial expenses (BEREZYUK 2009). Before SDW transportation by dustcarts to the place of their utilization, the waste loading is performed. According to approximate calculations, almost 2.7 thousand tone of fuel is yearly spent on the SDW loading by communal enterprises of Ukraine. "The Stockholm Convention on persistent organic pollutants" requires the SDW processing, collecting, transportation and storage in environmentally friendly manner. In particular, actual there is a problem of providing reducing the fuel consumption during SDW loading into a dustcart.

The analysis (SAVULYAK, BEREZYUK 2006) of developments in the field of SDW loading has shown that waste loading into the overwhelming majority of dustcarts is performed by means of a hydraulic drive of working organs. It has been found that the operation of SDW loading into a dustcart consists from turn of a lever and upturning of a container capture. The article (BEREZYUK 2013) provides a mathematical model of hydraulic drive of container upturning during SDW loading into dustcart as a substantially non-linear system of differential equations, that cannot be solved by known analytical methods in the permissible limits of error. Compared to the publications of previous authors, the scientific novelty of this article is as follows:

– a simplified mathematical model of hydraulic drive of container upturning during loading of solid domestic wastes into dustcart is proposed;

– approximated analytical time dependencies of pressure in a delivery pipe of hydrocylinder, angular velocity and the angle of container upturning are obtained;

– approximated duration dependence of container upturning from main parameters of hydraulic drive is detected;

– optimal values of the main parameters of working organs which ensure a minimum time of container upturning during loading of solid domestic wastes into a dustcart are defined.

The purpose of the study is to define optimal values of main parameters of working organs, which ensure the reduction of fuel consumption by means of time minimization of container upturning during SDW loading into a dustcart.

Methods

During the study such methods were used: imitative computer modeling, Laplace transform (SVESHNIKOV, TIHONOV 2005), decomposition into simple fractions. Table 1 shows the comparison of assumptions, taken into account during the development of full and simplified mathematical models.

Table 1
Comparison of assumptions, taken into account during the development of full and simplified mathematical models

Full mathematical model	Simplified mathematical model
Pressures at the pump output and at the hydrocylinder input are different	pressures at the pump output and at the hydro-cylinder input are same
Pressures in drain pipes are more than zero	pressures in drain pipes are around zero
Influence of the force of viscous friction on functioning of a hydraulic drive is essential	influence of the force of viscous friction on fun-ctioning of a hydraulic drive is unessential
Shoulders of appendix of efforts to executive organs take into account the change of angular velocity of the container	shoulders of appendix of efforts to executive or-gans do not take into account the change of angu-lar velocity of the container

Results and Discussion

Figure 1 presents a calculated scheme for a simplified mathematical model of hydraulic drive of container upturning during SDW loading into dustcart when using the back scheme of SDW loading by which such known brands of dustcarts are functioning: Norba N-Series (*Norba. Refuse Collection...* 1977), Schörling Olympus (*Schörling: Vehicle Technology...* 1990), Zoeller Magnum (*Zoeller* 1990, p. 8), Faun Europress (*Pressmullfahrzeuge Faun Europress* 1995) and others.

The following structural elements are marked on the scheme: *Cn* – container, *Cp* – capture, *L* – lever, *HC* – hydrocylinder, *HD* – hydrodistributor, *HP* – hydropump, *SV* – safety valve, *F* – filter, *T* – tank with working fluid, and also the main geometrical, kinematics and power parameters:

p_1, p_2, p_3, p_4 – pressures at the output of a hydropump, at the input of a hydrocylinder, at the output of a hydrocylinder and at the input of a filter, accordingly; W_1, W_2, W_3, W_4 – volumes of pipelines between the hydropump and the hydrodistributor, the hydrodistributor and the input of the hydrocylinder, the output of the hydrocylinder and the hydrodistributor, the hydrodistributor and the filter, Q_P – actual hydropump feeding, S_D – area of a passable hole of a hydrodistributor, S_F – surface area of a filter element, D, d – diameters of a piston and a rod, J – inertia moment of moving elements, G_C – weight of the container, R – rotation radius of moving elements, l_P – distance between rotation centers of the capture and the rod, h_C – altitude of the container, α – angle between the axes of a lever and the shoulder of a hydrocylinder, γ – angle which takes into account the position deflection of masses center, δ – angle between the shoulder of a capture and a horizontal, λ – inclination

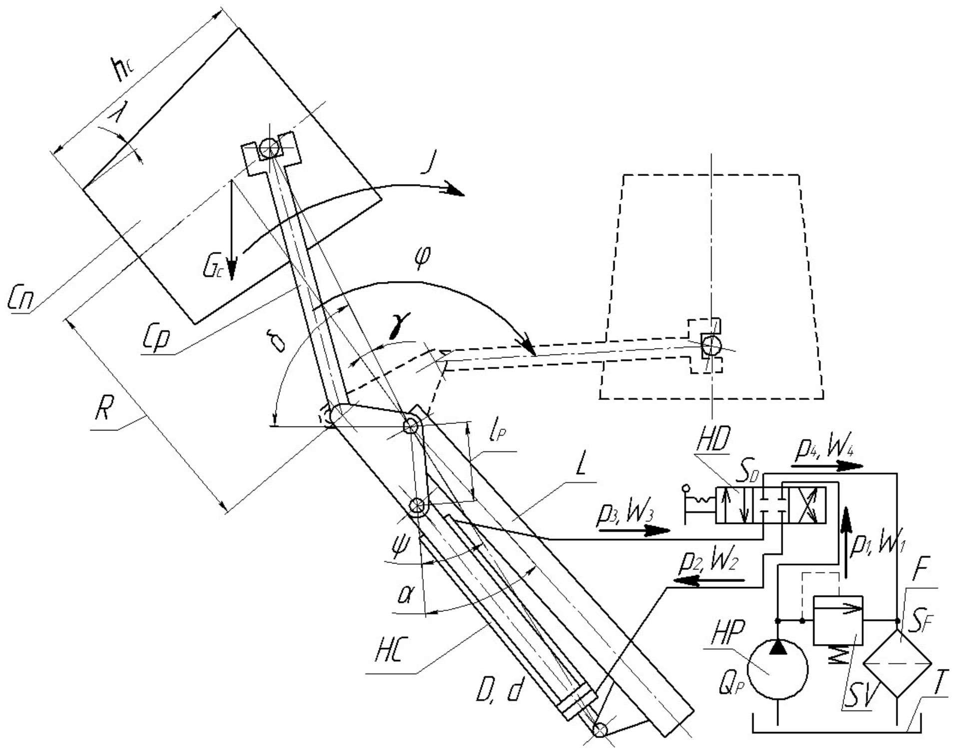

Fig. 1. Calculated scheme for a simplified mathematical model of hydraulic drive of container upturning during SDW loading into a dustcart (description in the text)

angle of a container wall, ψ – angle between the axis of a hydrocylinder shoulder and the axis which passes between the rotation centers of a capture and a hydrocylinder, φ – angle of capture rotation.

The analysis of the conducted studies of a full mathematical model (BEREZYUK 2013) has shown, that $p_1 \approx p_2 \approx p_{12}$, and the influence of pressure in drain pipes, forces influence of viscous friction on functioning of hydraulic drive is unessential.

Process of container upturning can be divided into two phases:

– phase of container turning to equilibrium position ($\delta + \varphi - \lambda \leq \pi/2$);

– phase of SDW emptying from the container into a basket of a dustcart ($\delta + \varphi - \lambda > \pi/2$).

Thus, the system of differential equations of full mathematical model turns into such systems of differential equations, which correspond to the phases, accepted above.

Phase of container turning to equilibrium position can be described by the system of differential equations:

$$\begin{cases} Q_P = \dot{\varphi} S_{C1} l_P \sin(\varphi + \psi) + \sigma p_{12} + K W_{12} \dot{p}_{12} & (1) \\ p_{12} S_{C1} l_P \sin(\varphi + \alpha) = J\ddot{\varphi} + GR\cos(\varphi + \delta - \gamma) & (2) \end{cases}$$

where:
$W_{12} = W_1 + W_2$.

For linearization of trigonometric functions of the system of differential equations we will introduce replacements:

$$\cos(\varphi + \delta - \gamma) \approx \cos(\omega_0 t + \delta - \gamma) \tag{3}$$

$$\sin(\varphi + \alpha) \approx \sin(\bar{\varphi}_1 + \alpha) \tag{4}$$

$$\sin(\varphi + \psi) \approx \sin(\bar{\varphi}_1 + \psi) \tag{5}$$

where:
$\omega_0 \approx \dfrac{Q_P}{2S_{C1}R}$ – average value of angular velocity of a container upturning at a first approximation;

(6)

$S_{C1} = \pi(D^2 - d^2)/4$ – area of a rod cavity of a hydrocylinder;
$\bar{\varphi}_1 = (\pi/2 + \lambda - \delta)/2$ – average value of the angle upturning of a container at a first approximation for the first phase.

Therefore, a simplified mathematical model of hydraulic drive of the first phase of container upturning during SDW loading into a dustcart looks like this:

$$\begin{cases} Q_P = 2\omega S_{C1} l_P \sin(\bar{\varphi}_1 + \psi) + \sigma p_{12} + K W_{12} \dot{p}_{12} & (7) \\ p_{12} S_{C1} l_P \sin(\bar{\varphi}_1 + \alpha) = J\dot{\omega} + GR\cos(\omega_0 t)\cos(\delta - \gamma) - GR\sin(\omega_0 t)\sin(\delta - \gamma) & (8) \end{cases}$$

where:
$\omega = \dot{\varphi} \neq \text{const.}$ – instantaneous value of angular velocity of container upturning.

For further study of a simplified mathematical model we use Laplace transform (SVESHNIKOV, TIHONOV 2005), according to which we get:

$$\begin{cases} Q_P/S = \Omega(s)2S_{C1}l_P \sin(\bar{\varphi}_1 + \psi) + P(s)\sigma + P(s)sKW_{12} & (9) \\ P(s)S_{C1}l_P \sin(\bar{\varphi}_1 + \alpha) = \Omega(s)sJ + \dfrac{sGR\cos(\delta - \gamma)}{s^2 + \omega_0^2} - \dfrac{\omega_0 GR\sin(\delta - \gamma)}{s^2 + \omega_0^2} & (10) \end{cases}$$

Substituting equation (10) into equation (9), we get

$$\Omega_1(s) = \frac{-b_3s^3 + b_2s^2 - b_1s + b_0}{s(s^2 + \omega_0^2)(a_2s^2 + a_1s + a_0)} \tag{11}$$

where:
$a_2 = KW_{12}J$; $a_1 = \sigma J$; $a_0 = 2S_{C1}^2 l_P^2 \sin(\bar{\varphi}_1 + \alpha)\sin(\bar{\varphi}_1 + \psi)$; $b_3 = KW_{12}GR\cos(\delta - \gamma)$; $b_2 = Q_PS_{C1}l_P\sin(\bar{\varphi}_1 + \alpha) - \sigma GR\cos(\delta - \gamma) + KW_{12}\omega_0 GR\sin(\delta - \gamma)$; $b_1 = \sigma\omega_0 GR$x $\sin(\delta - \gamma)$; $b_0 = Q_P\omega_0^2 S_{C1}l_P\sin(\bar{\varphi}_1 + \alpha)$

(12)

By the method of the expression (11) decomposition into simpler fractions after transformation to canonical form we get

$$\Omega_1(s) = A_1\frac{1}{s} + B_1\frac{s}{s^2 + \omega_0^2} + \frac{D_1}{a_2}\frac{s + a_1/(2a_2)}{[s + a_1/(2a_2)]^2 + (4a_0a_2 - a_1^2)/(4a_2^2} +$$
$$+ \frac{C_1}{\omega_0}\frac{\omega_0}{s^2 + \omega_0^2} + \frac{4E_1 - D_1a_1}{2\sqrt{4a_0a_2 - a_1^2}}\frac{\sqrt{4a_0a_2 - a_1^2}/(2a_2)}{[s + a_1/(2a_2)]^2 + (4a_0a_2 - a_1^2)/(4a_2^2)}, \tag{13}$$

where:
$A_1 = b_0/(a_0\omega_0^2)$; $B_1 = [(b_2\omega_0^2 - b_0)(a_0 - a_2\omega_0^2) - a_1\omega_0^2(b_3\omega_0^2 - b_1)]/\{\omega_0^2[(a_0 - a_2\omega_0^2)^2 + + a_1^2\omega_0^2]\}$; $C_1 = (b_3\omega_0^2 - b_1 + B_1a_1\omega_0^2)/(a_0 - a_2\omega_0^2)$; $D_1 = -a_2(A_1 + B_1)$; $E_1 = -b_3 + + D_1a_1/a_2 - C_1a_2$

(14)

We find the original of an image (13)

$$\omega_1(t) = A_1 + B_1\cos(\omega_0 t) + \frac{C_1}{\omega_0}\sin(\omega_0 t) + \frac{D_1}{a_2}e^{-\frac{a_1}{2a_2}t}\cos\left(\frac{\sqrt{4a_0a_2 - a_1^2}}{2a_2}t\right) +$$
$$+ \frac{4E_1 - D_1a_1}{2\sqrt{4a_0a_2 - a_1^2}}e^{-\frac{a_1}{2a_2}t}\sin\left(\frac{\sqrt{4a_0a_2 - a_1^2}}{2a_2}t\right) \tag{15}$$

After excluding minor coefficients of the expression (15) which have a higher order of smallness, and after taking into account the agreed designations according to (6), (12), (14), the angular velocity of container upturning during the first phase is presented by such equation:

$$\omega_1(t) \approx \frac{Q_P S_{C1} l_P \sin(\bar{\varphi}_1 + \alpha) - \sigma G R \cos(\delta - \gamma)}{2S_{C1}^2 l_P^2 \sin(\bar{\varphi}_1 + \alpha) \sin(\bar{\varphi}_1 + \psi)} \left[1 - e^{-\frac{\sigma}{2KW_{12}}t} \times \right. \tag{16}$$

$$\left. \times \cos\left(S_{C1} l_P \sqrt{\frac{2\sin(\bar{\varphi}_1 + \alpha)\sin(\bar{\varphi}_1 + \psi)}{KW_{12}J}}\, t\right)\right]$$

For angle definition of container upturning during the first phase we integrate the equation (16) and, taking into account the initial conditions $\varphi(0) = 0$, we get:

$$\varphi_1(t) = \frac{Q_P S_{C1} l_P \sin(\bar{\varphi}_1 + \alpha) - \sigma G R \cos(\delta - \gamma)}{2S_{C1}^2 l_P^2 \sin(\bar{\varphi}_1 + \alpha)\sin(\bar{\varphi}_1 + \psi)} \left\{ \frac{e^{-\frac{\sigma}{2KW_{12}}t}}{8KW_{12}S_{C1}^2 l_P^2 \sin(\bar{\varphi}_1 + \alpha)\sin(\bar{\varphi}_1 + \psi) + J\sigma^2} \times \right.$$

$$\times \left[2KW_{12}J\sigma\cos\left(S_{C1}l_P\sqrt{2\sin(\bar{\varphi}_1+\alpha)\sin(\bar{\varphi}_1+\psi)/(KW_{12}J)}\,t\right) - 4K^{1.5}W_{12}^{1.5}S_{C1}l_P \times \right. \tag{17}$$

$$\left.\left. \times \sqrt{2J\sin(\bar{\varphi}_1+\alpha)\sin(\bar{\varphi}+\psi)}\,\sin\left(S_{C1}l_P\sqrt{2\sin(\bar{\varphi}_1+\alpha)\sin(\bar{\varphi}_1+\psi)/(KW_{12}J)}\,t\right)\right] + t\right\}$$

After excluding minor coefficients of expression (17), which have a higher order of smallness, we receive a simplified equation of angle change of upturning container during the first phase:

$$\varphi_1(t) \approx \frac{Q_P S_{C1} l_P \sin(\bar{\varphi}_1 + \alpha) - \sigma G R \cos(\delta - \gamma)}{2S^2_{C1} l^2_P \sin(\bar{\varphi}_1 + \alpha)\sin(\bar{\varphi}_1 + \psi)}\, t \tag{18}$$

We define the duration of container upturning during the first phase by the equation (18):

$$t_1 \approx \frac{2S^2_{C1} l^2_P \sin(\bar{\varphi}_1 + \alpha)\sin(\bar{\varphi}_1 + \psi)}{Q_P S_{C1} l_P \sin(\bar{\varphi}_1 + \alpha) - \sigma G R \cos(\delta - \gamma)}\, \varphi \tag{19}$$

After placing the changes $\sin(\bar{\varphi}_1 + \alpha)\sin(\bar{\varphi}_1 + \psi) \approx \sin^2(\bar{\varphi}_1 + (\alpha + \psi)/2)$ and $\bar{\varphi}_1 \to \varphi_1(t)$ in the equation (16) we get:

$$\omega_1(t) \approx \frac{Q_P S_{C1} l_P \sin\left(\dfrac{Q_P}{2S_{C1}l_P \sin(\bar{\varphi}_1 + \psi)}\, t + \alpha\right) - \sigma GR\cos(\delta - \gamma)}{2S^2_{C1}l^2_P \sin^2\left(\dfrac{Q_P}{2S_{C1}l_P\sin(\bar{\varphi}_1 + \psi)}\, t + \dfrac{\alpha + \psi}{2}\right)} \times \tag{20}$$

$$\times\left[1 - e^{-\frac{\sigma}{2KW_{12}}t}\cos\left(S_{C1}l_P \sin\left(\frac{Q_P}{2S_{C1}l_P\sin(\bar{\varphi}_1 + \psi)}\, t + \frac{\alpha + \psi}{2}\right)\sqrt{\frac{2}{KW_{12}J}}\, t\right)\right]$$

After solving a system of equations (9, 10) relative to $P(s)$ after transformation to canonical form we get:

$$P_1(s) = A_{1p}\frac{1}{s} + \frac{B_{1p} - C_{1p} - F_{1p}}{KW_{12}}\frac{1}{s^2 + \sigma/(KW_{12})} - D_{1p}\frac{s}{s^2 + \omega^2_0} - \frac{E_{1p}}{\omega_0}\frac{\omega}{s^2 + \omega^2_0} - \frac{G_{1p}}{a_2}\times$$

$$\times\frac{s + a_1/(2a_2)}{[s + a_1/(2a_2)]^2 + (4a_0a_2 - a^2_1)/(4a^2_2)} - \frac{4H_{1p} - G_{1p}a_1}{2\sqrt{4a_0a_2 - a^2_1}}\times \tag{21}$$

$$\times\frac{\sqrt{4a_0a_2 - a^2_1}/(2a_2)}{[s + a_1/(2a_2)]^2 + (4a_0a_2 - a^2_1)/(4a^2_2)}$$

where:

$A_{1p} = (Q_P - 2A_1S_{C1}l_P\sin(\bar{\varphi}_1 + \psi))/\sigma$; $B_{1p} = -KW_{12}(Q_P - 2A_1S_{C1}l_P\sin(\bar{\varphi}_1 + \psi))/\sigma$,
$C_{1p} = -2S_{C1}l_PKW_{12}(B_1\sigma - C_1KW_{12})\sin(\bar{\varphi}_1 + \psi)/(\sigma^2 + K^2W^2_{12}\omega^2_0)$,
$D_{1p} = 2S_{C1}l_P(B_1\sigma - C_1KW_{12})\sin(\bar{\varphi}_1 + \psi)/(\sigma^2 + K^2W^2_{12}\omega^2_0)$,
$E_{1p} = 2S_{C1}l_P(B_1KW_{12}\omega^2_0 + C_1\sigma)\sin(\bar{\varphi}_1 + \psi)/(\sigma^2 + K^2W^2_{12}\omega^2_0)$,
$H_{1p} = 2S_{C1}l_P(E_1(a_1KW_{12} - a_2) - D_1a_0KW_{12})\sin(\bar{\varphi}_1 + \psi)]/(a_1KW_{12}\sigma - a_2\sigma^2 - a_0K^2W^2_{12})$,
$F_{1p} = 2S_{C1}l_PKW_{12}(D_1\sigma - E_1KW_{12})\sin(\bar{\varphi}_1 + \psi)/(a_1KW_{12}\sigma - a_2\sigma^2 - a_0K^2W^2_{12})$,
$G_{1p} = 2S_{C1}l_Pa_2(D_1\sigma - E_1KW_{12})\sin(\bar{\varphi}_1 + \psi)/(a_1KW_{12}\sigma - a_2\sigma^2 - a_0K^2W^2_{12})$
(22)

We find the original of an image (21):

$$p_1(t) = A_{1p} + \frac{B_{1p} - C_{1p} - F_{1p}}{KW_{12}}e^{-\frac{\sigma}{2KW_{12}}t} - D_{1p}\cos(\omega_0t) - \frac{E_{1p}}{\omega_0}\sin(\omega_0t) -$$

$$-\frac{G_{1p}}{a_2}e^{-\frac{\sigma}{2a_2}t}\cos\left(\frac{\sqrt{4a_0a_2 - a^2_1}}{2a_2}\, t\right) - \frac{4H_{1p} - G_{1p}a_1}{2\sqrt{4a_0a_2 - a^2_1}}e^{-\frac{\sigma}{2a_2}t}\sin\left(\frac{\sqrt{4a_0a_2 - a^2_1}}{2a_2}\, t\right) \tag{23}$$

After excluding minor coefficients of expression (23) which have a higher order of smallness, and after considering the agreed designations according to (6), (12), (14), (22) and the initial condition $p(0) = 0$, the pressure in a delivery pipe of a hydrocylinder can be described by the equation:

$$p_1(t) \approx \frac{GR}{S_{C1}l_P \sin\left(\dfrac{Q_P}{2S_{C1}l_P \sin(\bar{\varphi}_1 + \psi)} t + \alpha\right)} + \frac{Q_P}{S_{C1}l_P \sin\left(\bar{\varphi}_1 + \dfrac{\alpha + \psi}{2}\right)} \times$$
$$\times \sqrt{\frac{J}{2KW_{12}}}\, e^{-\frac{\sigma}{2KW_{12}} t} \sin[S_{C1}l_P \sin(\bar{\varphi}_1 + (\alpha + \psi)/2)\sqrt{2/(KW_{12}J)}t + \tag{24}$$
$$- \arcsin(\sqrt{2KW_{12}J}\sigma GR \cos(\delta - \gamma) \sin(\bar{\varphi}_1 + (\alpha + \psi)/2)/(Q_P \sin \alpha))]$$

Phase of SDW emptying from the container into a basket of the dustcart can be described by a system of differential equations:

$$\begin{cases} Q_P = 2\dot{\varphi}S_{C1}l_P \sin(\varphi + \psi) + \sigma p_{12} + KW_{12}\dot{p}_{12} & (25) \\ p_{12}S_{C1}l_P \sin(\varphi + \alpha) = J\ddot{\varphi} + (GR - V_C\rho_w x(1 + 2\mathrm{tg}\lambda)Rg/h_C)\cos(\varphi + \delta - \gamma) & (26) \\ g \sin(\varphi + \delta - \lambda) = \ddot{x} + f_w g \cos(\varphi + \delta - \lambda) & (27) \end{cases}$$

For linearization of trigonometric functions in the system of differential equations we will introduce the replacements:

$$\cos(\varphi + \delta - \gamma) \approx \cos(\omega_0 t + \delta - \gamma) \approx \cos(\bar{\varphi}_2 + \delta - \gamma) \tag{28}$$

$$\sin(\varphi + \delta - \gamma) \approx \sin(\omega_0 t + \delta - \gamma) \tag{29}$$

$$\sin(\varphi + \alpha) \approx \sin(\bar{\varphi}_2 + \alpha) \tag{30}$$

$$\sin(\varphi + \psi) \approx \sin(\bar{\varphi}_2 + \psi) \tag{31}$$

where:
$\bar{\varphi}_2 = 0.75\pi + 1.5\lambda - \delta$ – average value of the angle of container upturning at a first approximation for the second phase.

Therefore a simplified mathematical model of a hydraulic drive of the second phase of container upturning during SDW loading into a dustcart looks like this:

$$\begin{cases} Q_P = 2\omega S_{C1} l_P \sin(\bar{\varphi}_2 + \psi) + \sigma p_{12} + KW_{12}\dot{p}_{12} & (32) \\ p_{12} S_{C1} l_P \sin(\bar{\varphi}_2 + \alpha) = J\dot{\omega} + GR\cos(\omega_0 t)\cos(\delta - \gamma) - \\ \quad - GR\sin(\omega_0 t)\sin(\delta - \gamma) - V_C \rho_w x(1 + 2\mathrm{tg}\lambda)\, Rg\cos(\bar{\varphi}_2 + \delta - \gamma)/h_C & (33) \\ g\sin(\omega_0 t + \delta - \lambda) = \ddot{x} + f_w g\cos(\omega_0 t + \delta - \lambda) & (34) \end{cases}$$

Solving a simplified mathematical model of the second phase in a similar way to solution the of a simplified mathematical model of the first phase, we get:

$$\omega_2(t) \approx \frac{Q_P S_{C1} l_P \sin(\bar{\varphi}_2 + \alpha) - \sigma GR\cos(\delta - \gamma)}{2S_{C1}^2 l_P^2 \sin(\bar{\varphi}_2 + \alpha)\sin(\bar{\varphi}_2 + \psi)} + \left[\frac{2S_{C1}R}{Q_P}\sin\left(\frac{Q_P}{2S_{C1}R}t\right) - t\right] \times$$
$$\times \frac{\sigma V_C \rho_w (1 + 2\mathrm{tg}\lambda)\, R^2 g^2\, [\cos(\delta - \lambda) + f_w \sin(\delta - \lambda)]\cos(\bar{\varphi}_2 + \delta - \gamma)}{Q_P S_{C1} l_P^2 \sin(\bar{\varphi}_2 + \alpha)\sin(\bar{\varphi}_2 + \psi) h_C} \quad (35)$$

$$\varphi_2(t) = \frac{Q_P S_{C1} l_P \sin(\bar{\varphi}_2 + \alpha) - \sigma GR\cos(\delta - \gamma)}{2S_{C1}^2 l_P^2 \sin(\bar{\varphi}_2 + \alpha)\sin(\bar{\varphi}_2 + \psi)} t - \left[\frac{t^2}{2} + \frac{4S_{C1}^2 R^2}{Q_P^2}\left(\cos\left(\frac{Q_P}{2S_{C1}R}t\right) - 1\right)\right] \times$$
$$\times \frac{\sigma V_C \rho_w (1 + 2\mathrm{tg}\lambda)\, R^2 g^2\, [\cos(\delta - \lambda) + f_w \sin(\delta - \lambda)]\cos(\bar{\varphi}_2 + \delta - \gamma)}{Q_P S_{C1} l_P^2 \sin(\bar{\varphi}_2 + \alpha)\sin(\bar{\varphi}_2 + \psi) h_C} \quad (36)$$

$$t_2 \approx \frac{2S_{C1}^2 l_P^2 \sin(\bar{\varphi}_2 + \alpha)\sin(\bar{\varphi}_2 + \psi)}{Q_P S_{C1} l_P \sin(\bar{\varphi}_2 + \alpha) - \sigma GR\cos(\delta - \gamma)}\varphi - \frac{2l_P}{Rg} \times$$
$$\times \sqrt{\frac{-Q_P S_{C1} l_P^2 \sin(\bar{\varphi}_2 + \alpha)\sin(\bar{\varphi}_2 + \psi) h_C \varphi}{\sigma V_C \rho_w (1 + 2\mathrm{tg}\lambda)\, R^2 g^2\, [\cos(\delta - \lambda) + f_w \sin(\delta - \lambda)]\cos(\bar{\varphi}_2 + \delta - \gamma)}} \quad (37)$$

$$p_2(t) \approx \frac{V_C \rho_w (1 + 2\mathrm{tg}\lambda)\, R^2 g^2 \cos(\bar{\varphi}_2 + \delta - \gamma)}{16 Q_P^2 l_P h_C \sigma \sin(\bar{\varphi}_2 + \alpha)} \{S_{C1} R\sigma[\sin(\delta - \lambda) - f_w \cos(\delta - \lambda)] \times$$
$$\times [\cos(Q_P/(2S_{C1}R)t) - 1] - 8KW_{12} Q_P [\cos(\delta - \lambda) + f_w \sin(\delta - \lambda)]\} \quad (38)$$

The dynamics of container upturning can be approximately described on the basis of the obtained functional dependencies for separate phases as follows:

$$\omega(t) = \begin{cases} \omega_1(t), \text{ for } \varphi \le \pi / 2 - \delta + \lambda \\ \omega_2(t), \text{ for } \varphi > \pi / 2 - \delta + \lambda \end{cases} \tag{39}$$

$$\varphi(t) = \frac{Q_P S_{C1} l_P \sin(\bar{\varphi} + \alpha) - \sigma G R \cos(\delta - \gamma)}{2 S_{C1}^2 l_P^2 \sin(\bar{\varphi} + \alpha) \sin(\bar{\varphi} + \psi)} t - \left[\frac{t^2}{2} + \frac{4 S_{C1}^2 R^2}{Q_P^2}\left(\cos\left(\frac{Q_P}{2 S_{C1} R} t\right) - 1\right)\right] \times$$

$$\times \mathbf{1}\left(\varphi - \frac{\pi}{2} + \delta - \lambda\right) \frac{\sigma V_C \rho_w (1 + 2\mathrm{tg}\lambda) R^2 g^2 [\cos(\delta - \lambda) + f_w \sin(\delta - \lambda)] \cos(\bar{\varphi}_2 + \delta - \gamma)}{Q_P S_{C1} l_P^2 \sin(\bar{\varphi}_2 + \alpha) \sin(\bar{\varphi}_2 + \psi) h_C} \tag{40}$$

$$p(t) = \begin{cases} p_1(t), \text{ for } \varphi \le \pi / 2 - \delta + \lambda \\ p_2(t), \text{ for } \varphi > \pi / 2 - \delta + \lambda \end{cases} \tag{41}$$

where:
$\bar{\varphi} = (\bar{\varphi}_1 + \bar{\varphi}_2)/2$ – average value of the angle of container upturning at a first approximation.

Comparison of the results, obtained with the use of full and simplified mathematical models of a hydraulic drive of container upturning during SDW loading into a dustcart, as well as by means of the equations, obtained in the result of the analytical solution of a simplified model is presented in Figure 2.

When using the equations (19), (37), such duration dependence of container upturning during SDW loading into a dustcart from the main parameters of its hydrodrive is obtained.

$$t = \begin{cases} t_1(\varphi), \text{ for } \varphi \le \pi / 2 - \delta + \lambda \\ t_2(\varphi), \text{ for } \varphi > \pi / 2 - \delta + \lambda \end{cases} \tag{42}$$

When comparing the duration of container upturning, obtained with the use of a full mathematical model and equations (39, 40, 41), obtained in the result of analytical solution of a simplified mathematical model of hydraulic drive of container upturning during SDW loading into a dustcart, an error makes up about 10% at the beginning of motion and reduces to about 2–5% at the end of motion in comparison with the full mathematical model, that is acceptable for the execution of previous project calculations. If necessary, the values of the main parameters can be specified at the final stage of projecting by means of a full mathematical model.

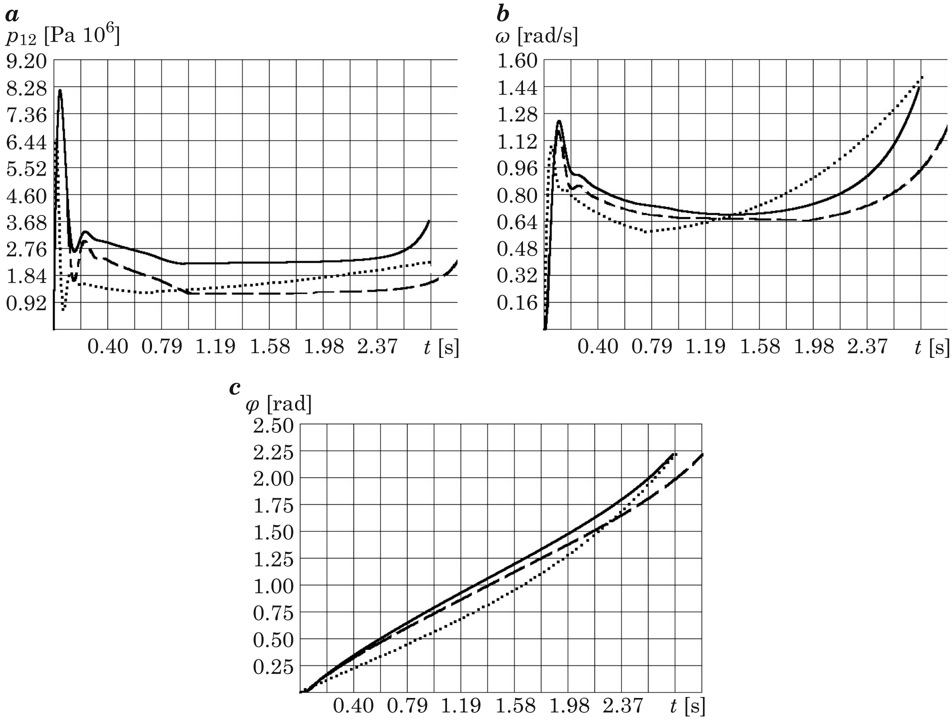

Fig. 2. Comparison of results, obtained with the use of full (——) and simplified (- - -) mathematical models of hydraulic drive of container upturning during SDW loading into a dustcart, as well as by means of equations obtained as result of its analytical solution (·····): *a* – change of pressure in the hydrocylinder, *b* – change of angular velocity, *c* – change of rotation angle

The obtained regression equation (42) allows to approximately define the duration of container upturning during SDW loading into a dustcart that can be used during performing of project calculations of new dustcarts constructions without the necessity of studying a full mathematical model of hydrodrive of its working organs, and also during the optimization of main parameters of a hydraulic drive.

Figure 3 shows the duration dependence of container upturning t from the distance between rotation centers of a capture and a rod l_p and the angle between axes of a lever and a shoulder of a hydrocylinder α, built according to the regression equation (42).

Comparison of power and speed characteristics for basic and optimal values of the main parameters of working organs of container upturning during SDW loading into a dustcart is presented in Figure 4. Reducing of the process duration of container upturning is accompanied by increased pressure in a delivery pipe of a hydrocylinder (Fig. 4a) and by increased angular velocity of container upturning (Fig. 4b).

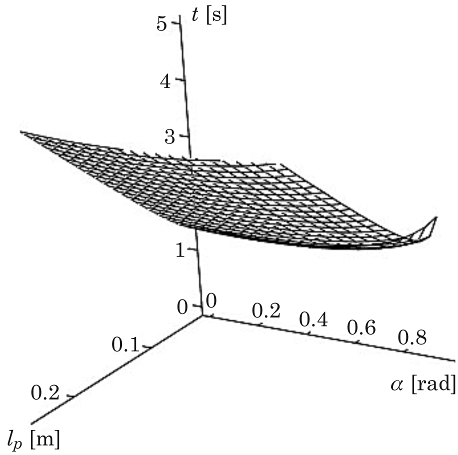

Fig. 3. The duration dependence of container upturning t from distance between rotation centers of a capture and a rod l_p and from the angle between axes of a lever and a shoulder of a hydrocylinder α

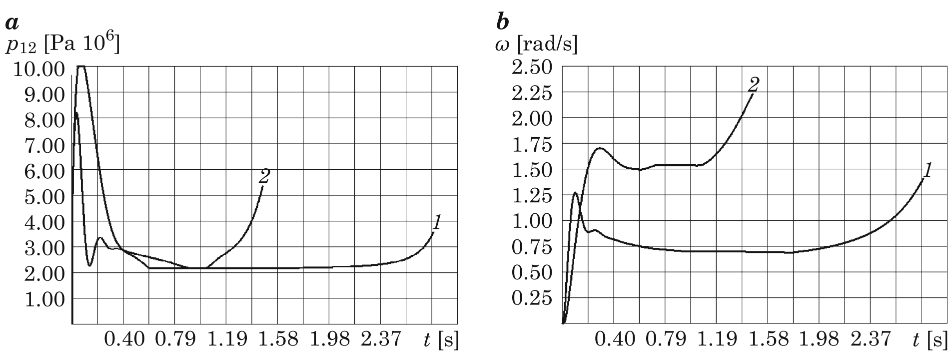

Fig. 4. Comparison of power (*a*) and speed (*b*) characteristics for basic (*1*) and optimal (*2*) values of main parameters of working organs of container upturning during SDW loading into a dustcart

By means of dependence (42) in MathCAD environment, the optimal values of the distance between the rotation centers of the capture and the rod $l_{p.\mathrm{opt}}$ = 38 mm and the angle between the axes of a lever and a shoulder of a hydrocylinder α_{opt} = 11° for which duration of container upturning will be minimal $t_{\min}$ = 1.468 s are defined, that can be used for process intensification of SDW loading into a dustcart with the reduction purpose of fuel consumption.

We define the yearly fuel savings for public utilities of Ukraine by the formula:

$$Q_S = q_{ro}K_cN_d\left(\frac{t_c}{t_c - n_c(t_{uc.b} - t_{uc.\min})/3{,}600} - 1\right) = 10 \cdot 1276 \cdot 4{,}100 \times \\ \times \left(\frac{1.89}{1.89 - 21(2{,}755 - 1.468)/3{,}600} - 1\right) = 208{,}690(l) \approx 209 \text{ [tons]} \quad (43)$$

where:

q_{ro} – rate of fuel consumption during one working cycle of SDW collecting into a basket of a dustcart [l],

K_c – yearly quantity of working cycles of a dustcart [units],

N_d – dustcarts quantity of public utilities of Ukraine [units],

t_c – duration of a working cycle of SDW collecting into a basket of a dustcart [h],

n_c – quantity of containers with SDW, that can be loaded into a basket of a dustcart per one cycle [units],

$t_{uc.b}$, $t_{uc.\min}$ – duration of container upturning for basic and optimal values of the main parameters of working organs, respectively [s].

Conclusions

1. For performing of project calculations of new constructions of dustcarts the approximate analytical time dependencies of pressure in a delivery pipe of a hydrocylinder, angular velocity and angle container upturning on the basis of a proposed simplified mathematical model of a hydraulic drive of container upturning during loading of solid domestic wastes into a dustcart are obtained.

2. Approximate duration dependence of container upturning from the main parameters of hydraulic drive is detected, on the basis of which the optimal values of distance between the rotation centers of a capture and a rod $l_{P.\text{opt}}$ = 38 mm and the angle between the axes of a lever and a shoulder of a hydrocylinder α_{opt} = 11° are defined, for which the duration of container upturning will be minimal $t_{\min}$ = 1.468 s, that can be used for intensification of the loading process of solid domestic wastes with the reduction purpose of fuel consumption at 209 tons/year.

References

BEREZYUK O.V. 2009. *Mathematical modelling of forecasting of the volumes forming of solid domestic waste and the areas of polygons and dumps in Ukraine.* Modern Technologies, Materials and Constructions in Civil Engineering, 2: 88–91.

BEREZYUK O.V. 2013. *Mathematical modeling of the dynamic of hydraulic drive of working organs of container upturning during solid domestic waste loading into the dustcart.* Herald of Khmelnytskyi national university. Technical Sciences, 5: 60–64.

Norba. Refuse Collection Vehicle Kl-10. 1977. Blomstermala, 3–4.

Pressmullfahrzeuge Faun Europress. 1995. Osterholz-Scharmbeck, 2–3.

SAVULYAK V.I., BEREZYUK O.V. 2006. *Technical support of collecting, transportation and preparations of solid domestic wastes to processing*. Monograph. Universum-Vinnytsia, Vinnytsia.

Schörling: Vehicle Technology for the Environment. 1990. Hannover.

SVESHNIKOV A.G., TIHONOV A.N. 2005. *Functions theory of a complex variable*. Fizmatlit, Moscow.

Zoeller. 1990. Stockerau.