Technical Sciences, 2021, 24, 157–170

UNIWERSYTET WARMIŃSKO-MAZURSKI W OLSZTYNIE

Yearbook peer-reviewed scientific journal
ISSN 1505-4675
e-ISSN 2083-4527
TECHNICAL SCIENCES
Homepage: https://czasopisma.uwm.edu.pl/index.php/ts/

DOI: https://doi.org/10.31648/ts.6914

MODIFICATION OF THE TOOTH GEOMETRY OF A POLYMER GEAR WITH A STRAIGHT TOOTH LINE TO ADJUST THE TORQUE TRANSMISSION CAPABILITY IN ONE DIRECTION ONLY

Piotr Strojny

ORCID: 0000-0001-8593-8548
Faculty of Mechanical Engineering and Aeronautics
Rzeszow University of Technology

Received 6 July 2021, accepted 28 September 2021, available online 10 October 2021.

K e y w o r d s: gears, tooth strength, tooth geometry modification, topological optimization, kinetic simulations.

A b s t r a c t

This paper describes a methodology for modifying a tooth for the ability to transmit torque in one direction only. It presents two methods (analytical and numerical) of tooth mass reduction while maintaining functional features of the whole transmission. The results of the above mentioned methodology are presented on the example of a mass-produced transmission.

Introduction

A significant percentage of devices incorporating gears are based on the principle of torque transmission in one direction only. Consequently, the gears in them use only one side of the tooth for direct torque transmission. This situation raises the important question of whether, and to what extent, the teeth of such gears can be modified in terms of the removal of unnecessary material, finishing treatments such as grinding, and heat treatments on the side of the tooth that

Correspondence: Piotr Strojny, Katedra Inżynierii Lotniczej i Kosmicznej, Politechnika Rzeszowska, al. Powstańców Warszawy 8. 35-959 Rzeszów, phone: 692 047 962, e-mail: pstrojny@prz.edu.pl

does not directly transmit torque. The paper mainly focuses on the first aspect of modification i.e. weight reduction, being the most measurable criterion in terms of numerical analysis.

After conducting a literature analysis of classic items (DZIAMA et al. 1995, HOMIK, POŁOWNIAK 2012, KURMAZ, KURMAZ 2016, MAZANEK 2012, OCHĘDUSZKO 1985), where the authors describe the modification of typical tooth geometries based on involutes and epicycloids as well as modern items (KOLLEK et al. 2015, OSIŃSKI et al. 2018, OSIŃSKI 2017) where the authors experiment with new contours, e.g. with polvolwent (OSIŃSKI 2017). It can be concluded that the subject matter discussed in the article is still valid and should be the subject of further research. It is worth to mention that the literature analysis did not come across any items that directly refer to the modification of the tooth geometry made of polymeric materials.

Research object

In this paper, a polymer transmission of Zelmer Diana 886.8 MP mincer (Fig. 1) was used for analysis (Zelmer. 2021). Using reverse engineering (DZIUBEK 2018, KIŃSKI, SOBIESKI 2020, RATAJCZYK 2005, SHAH et al. 2019, SUN et al. 2019), the last gear speed was scanned from the output torque side – see Figure 1. Then, detailed measurements of the geometry of the scanned teeth were made, and on this basis the parameters of the mating pairs of gears were estimated. Some simplification was also made by replacing the angled teeth with straight teeth. DuPont™ company (Dupont. 2021), the supplier of polymers for Zelmer company, provided information about the material of which the above mentioned transmission is made. All information is summarized in Tables 1 and 2.

Table 1

Transmission parameters

Name	Modulus	Number of teeth	Width of rim	Pitch diameter	Pressure angle	Apical play	Lateral play	Tooth apex rounding	Output torque	Axial distance of mating gears	Tooth-form factor y	Tooth-form factor x
Unit	mm	–	mm	mm	deg	mm	mm	mm	Nm	mm	–	–
Pinion	2	9	20	18	20	0.5	0.08	0.3	3.575	48	1	0
Gear	2	39	18	78								

Source: based on Zelmer (2021).

Table 2

Material data for pairs of gears

Name	Young's modulus	Poisson number	Yield strength	Hardness H 358/30	Density
Unit	MPa	–	MPa	MPa	g/cm^3
Derlin 500P NC010	300	0.37	70.5	192	1.42
Standard	ISO 179	ISO 527	ISO 527-2	ISO 2039-1	ISO 1183

Source: based on Dupont (2021).

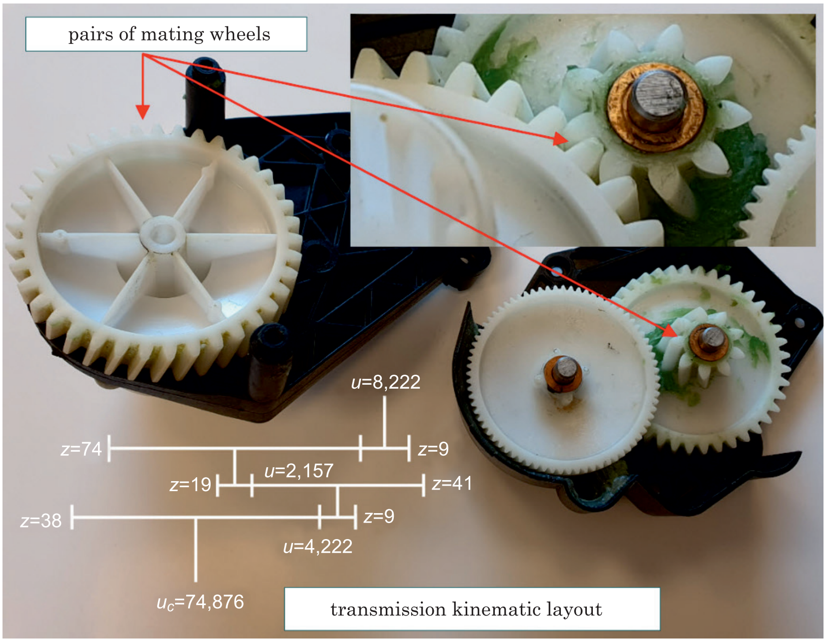

Fig. 1. Transmission from ZELMER Diana 886.8 MP mincer Transmission kinematic system
Source: based on Zelmer (2021).

According to the kinematic diagram (Fig. 1) and the data from Tables 1 and 2 using the relationships (1) and (2) (DZIAMA et al. 1995, OCHĘDUSZKO 1985), stress values at the tooth base and contact stresses (Hertz formula) were estimated:

$$\sigma_g \geq \frac{6\,F\,h}{b\,s^2} \tag{1}$$

$$\mathrm{p_{max.}} \geq \sqrt{\frac{2}{\left(\frac{1}{\mathrm{E}}+\frac{1}{\mathrm{E}}\right)\pi(1-\nu^2)\sin(\alpha)\cos(\alpha)}}\sqrt{\frac{F_{\mathrm{calc}}}{b\,d_1}\left(1+\frac{1}{u}\right)} \tag{2}$$

where:

σ_g – allowable bending stresses,
F – circumferential force from bending torque,
h – tooth height,
s – tooth thickness at the base, estimated on the basis of the numerical model (Fig. 3),
$p_{max.}$ – allowable stress under pressure,
E – Young's modulus of Derlin 500P NC010,
ν – Poisson's ratio for Derlin 500P NC010,
a – pressure angle,
F_{calc} – resultant of circumferential force equal to $F \cos(a)$,
u – gear ratio in the analysed part of the transmission.

The obtained values of stresses at the tooth base and at contact surfaces give safety factors of 2.1 and 3.9 respectively, which permits to assume that the analysed geometry of the transmission has been selected correctly.

Tooth model

The numerical model of the transmission's teeth was developed based on the chiseling method (CHERNETS 2019, POŁOWNIAK et al. 2020, TWARDOCH 2014). See Figure 2 for a description of the individual diameters and curves *3* and *4*.

The graphic interpretation of the above-mentioned one is described in more detail in DZIAMA et al. (1995, p. 26-30, 43-48) and OCHĘDUSZKO (1985, p. 61-63).

Ordinary involute equation – tooth side:

$$\begin{aligned} x_b &= r_b(\sin\phi - \hat{\varphi}\cos\phi) \\ y_b &= r_b(\cos\phi + \hat{\varphi}\sin\phi) \end{aligned} \tag{3}$$

Epicycloid equations – transition curve at tooth base:

$$\begin{aligned} x_p &= a\sin(\phi) - r_{a2}\sin\left(\left(1+\frac{r_1}{r_2}\right)\phi\right) \\ y_p &= a\cos(\phi) - r_{a2}\cos\left(\left(1+\frac{r_1}{r_2}\right)\phi\right) \end{aligned} \tag{4}$$

where:

x_b, y_b – coordinates of the points of the involute describing the side of the tooth,
x_p, y_p – coordinates of the epicycloid points describing the transition curve at the base,

ϕ – angle of deflection of radial coordinate,
$\hat{\phi}$ – arc measure of the angle of deflection of the radial coordinate,
r_b – basic radius,
a – axial distance of mating gears,
$r_{a1,2}$ – radii of tooth heads,
$r_{1,2}$ – pitch radii of gears.

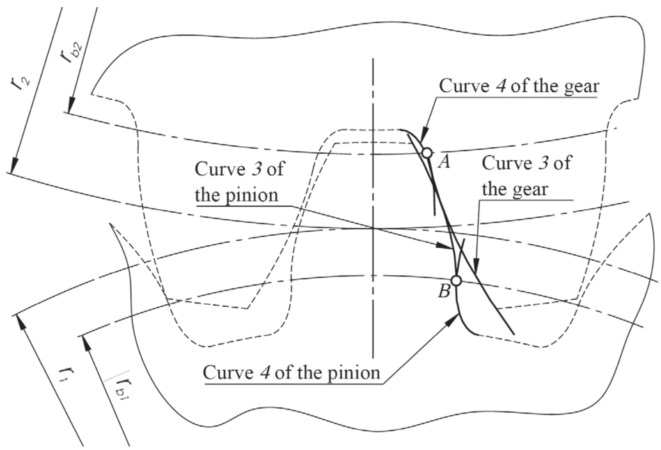

Fig. 2. Numerical model of curves describing transmission teeth geometry: *A* – point connecting the base transition curve *4* with the curve describing the side of the tooth of the gear *3*, *B* – point connecting the base transition curve *4* with the curve describing the side of the tooth of the pinion *3*

An important factor in building a numerical model of a pinion tooth that will subsequently be subjected to loading is to correctly determine the pressure number and the areas in which the tooth is in single or double contact with the mating wheel. This allows to determine the tooth position that is least favourable in terms of the load to be transferred. Figure 3 shows two typical worst-case tooth positions. The first area is the variant in which the tooth is loaded with 1/2 of the peripheral force but on the longest arm (h_1), while the second area is the variant in which the tooth is loaded with the total peripheral force but on the shorter arm (h_2). Depending on the gear parameters, one of the above variants will result in higher stresses at the base of the tooth. Due to the large number of variables in a gear, the correct process of estimating the above mentioned areas must be done individually for each gear under consideration. Such an analysis can be carried out by means of modern CAD programs that allow kinetic simulations of gear mating (KROL, SOKOLOV 2021, PETR, DYNYBYL 2014) and software dedicated to such tasks as KISSsoft (KISSsoft. 2021), KIMoS (KLINGELNBERG. 2021).

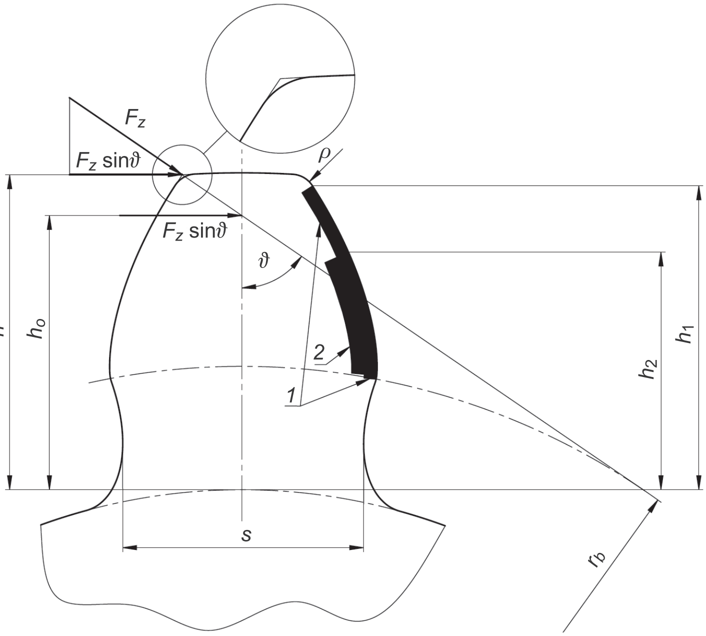

Fig. 3. Contact areas for one-pair (*2*) and two-pair (*1*) contact

The pressure number for the case under consideration was determined using CAD software in the dynamic simulation module. This allowed to determine how many pairs of teeth are in mesh for one complete cycle. For the analyzed transmission the pressure number was 1.31 for the test parameters: cycle 40 deg, number of transition steps 200/cycle.

According to OCHĘDUSZKO (1985, p. 354-356), the relationship (1) can also be presented in the form of equation (5). The author of the above-mentioned item states that the force F_z is distributed over one pair of teeth or into two pairs of teeth. For the calculations, however, it accepts only the second case, assuming the reduction of the force F_z by dividing it by ε_α (tooth contact ratio) (6) (DZIAMA et al. 1995, p. 63-65). At the same time stating that you are not making too much of a mistake:

$$\sigma_g \geq \frac{6\, F_z \sin\vartheta\, h_o}{b\, s^2} \tag{5}$$

$$\varepsilon_\alpha = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a_w \sin\alpha}{\pi\, m \cos\alpha} \tag{6}$$

where:

$r_{b1,2}$ – basic radius,

$r_{a1,2}$ – radii of tooth heads,

a_w – axial distance of mating gears,
a – pressure angle,
m – modulus,
h_0 – bending moment arm,
h – tooth depth.

For the analysed gear, in accordance with DZIAMA et al. (1995), ε_a was 1.36, which is a similar value to the result obtained from the numerical simulation (1.31). The obtained difference may result from the fact that in item DZIAMA et al. (1995) the rounding at the tooth apex was not taken into account (Fig. 3). However, with regard to the maximum bending stress at the tooth base, according to items OCHĘDUSZKO (1985) and DZIAMA et al. (1995) formulas (5) and (6), the maximum bending stress was σ_g = 25.64 MPa. This value differs from the values obtained from the numerical analysis, which for the arm h_1 and 1/2 the value of the force F_z amounted to σ_g = 19.29 MPa and for h_2 and the full value of the force F_z, σ_g = 29.62 MPa. Taking into account the greater stresses, the obtained difference is slightly more than 13%. It may result mainly from the aforementioned rounding (ρ = 0.3 mm) at the tooth apex, which occurs in the numerical analysis and is not included in the analytical formulas (5) and (6).

It is evident from the analyses presented above that the number of buttresses is of significant importance. Its growth reduces the stresses at the base. For the considered case, achieving ε_a equal to 2 would result in reaching σ_g = 25.64 MPa, that is lowering the stresses at the base by almost 4 MPa.

Tooth modification based on analytical calculations

After the numerical model of the tooth was made, the modification of its shape in terms of mass reduction due to the unidirectional nature of operation followed. A pinion tooth was used for the analysis due to the fact that it has a larger undercut at the base which leads to higher bending stresses. Based on relation (1) estimating the tooth thickness, graphs of tooth thickness minima versus its height {1}{2} (Fig. 4) were developed for two loading variants according to Figure 3. The effect of contact stresses is also considered in Figure 4. Based on Hertz's formula (7) (OCHĘDUSZKO 1985, p. 369-371, 401-413) and Bielajew point (8) (BERCZYŃSKI et al. 2016), the minimum area to be preserved on the lateral tooth surface {3} was determined:

$$a = 1.52 \sqrt{\frac{F_{\mathrm{calc}}}{b\,E} \frac{r_1\, r_2}{r_1 + r_2}} \tag{7}$$

$$y = 0.78\, a \tag{8}$$

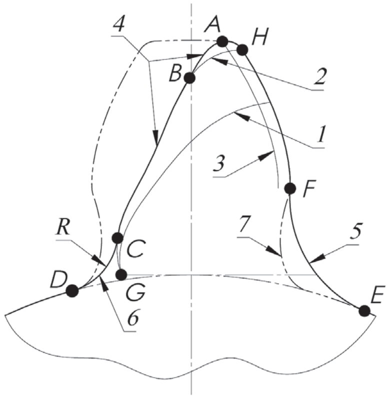

Fig. 4. Shape of the tooth after geometric modification; solid line – modified tooth shape, dashed line – base tooth shape

Due to the fact that the mating gear has undergone the same process of tooth shape modification, it was possible to replace the transition curve at the base of the pinion (epicycloid) by a classical arc tangent to the tooth working surface (involute) and the bottom of the tooth notch {5} arc |FE|. This resulted in a much smaller undercut and allowed us to model the side of the tooth responsible for the transmission of torque |HFE|. The side of the tooth on the side not carrying the torque {4} was determined from equation (1) taking into account the contact stresses (7) and (8). This allowed to obtain the tooth side based on equations (1), (3), (7) and (8), shown in Figure 4. In order to execute technologically the thus obtained tooth geometry it was necessary to replace the point *G* by the arc |CD| with radius {*R*} of the tool making the notch. A commercially available ϕ2 cutter with a length of 20 mm was used due to the small tooth size relative to its width (Narzędzia profesjonalne EU. 2021). In addition, for reasons of the manufacturing process, the theoretical curve which should result from the resultant curves {2} and {3} should be replaced by curve |AB| tangent at point A to the rounding of the tooth apex. This allows to obtain the overall shape of the tooth {*a*}. The figure also shows the base shape of the tooth {*b*}.

Tooth modification based on topology optimization

Due to the fact that modern FEM programs allow to perform numerical optimization (topology optimization) of structural elements, it has been decided, based on available examples from literature (DEPTULA, PARTYKA 2018, LARSSON 2016, MUMINOVIC et al. 2020, SHAH et al. 2019, SUN et al. 2019), to perform such optimization for the tooth of the gear described. Note that for the simulation, the tooth was modified to remove the excessive undercut {7} and replaced with an arc {5} |FE| tangent to the side of the tooth at point *F* and the base arc (Fig. 4). The analysis was performed for simulation parameters Table 3. The results obtained are shown in Figure 5. Using granularity removal techniques (BRAUER 2002, BRECHER et al. 2009), the final pinion tooth shape obtained by topological optimization process of Figure 5*c* was developed.

Table 3

Boundary conditions

Nazwa	Typ	Unit
Mesh type	solid mesh	–
Jacobian points	16	–
Element size	0.2	mm
Tolerance	0.01	mm
Total number of elements	299559	–
Solver type	Intel Direct Sparse	–
Total number of nodes	55528	–
Maximum aspect ratio	38.412	–
Percentage of elements with an aspect ratio <3	99.7	–
Percentage of elements with an aspect ratio >10	0.0945	–
Goal type	best stiffness to weight ratio	–
Reduction	reduce weight by 14%	–
Model type	Linear Flexible Isotropic	–

Source: based on LARSSON (2016).

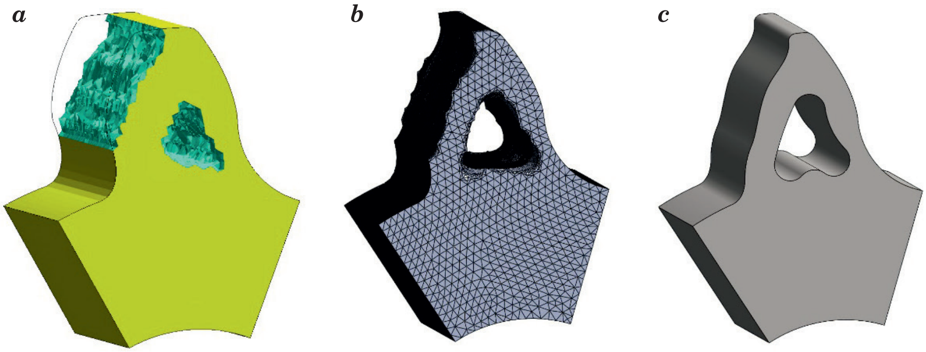

Fig. 5. Topological optimization results: *a* – view with areas to be removed, *b* – resulting model with granular structure, *c* – smoothed model
Source: based on LARSSON (2016).

Analysis of results

The following section summarizes the results obtained by modification based on analytical relations and numerical optimization with the basic tooth geometry. The front surface of the tooth was chosen for comparison because the length of the tooth did not change in either method. Table 4 shows all the results obtained along with the percentage weight reduction assigned to each method relative to the original tooth geometry.

Table 4

Summary of tooth mass reduction results for different modification methods

Method	Tooth face area [mm^2]	Percentage weight reduction [%]
Basic tooth shape	4,135.391	–
Tooth shape after analytical modification	3,207.696	22.433
Tooth shape after topological optimization	2,383.496	42.3635

Note that the tooth mass is only a part of the mass of the entire gear. Therefore, in order to present the results more vividly, Table 5 additionally shows by how much the mass of the analyzed gear decreased for the proposed modification methods.

Table 5

Summary of the results of reducing the mass of a gear for different modification methods

Method	Pinion wheel mass [mm^2]	Percentage mass reduction [%]
Basic tooth shape	4,135.391	–
Tooth shape after analytical modification	3,207.696	9.966
Tooth shape after topological optimization	2,383.496	18.919

Figure 6 contains the superimposed tooth geometries obtained by the three methods. After obtaining the final tooth shapes for the two methods described above, kinetic analysis of the mating was performed to check for the occurrence of possible collisions and the pressure number was re-estimated. The results of the analyses did not reveal any irregularities in mating and the pressure number did not change. Figure 7 shows a visualization of the obtained pinions for all methods.

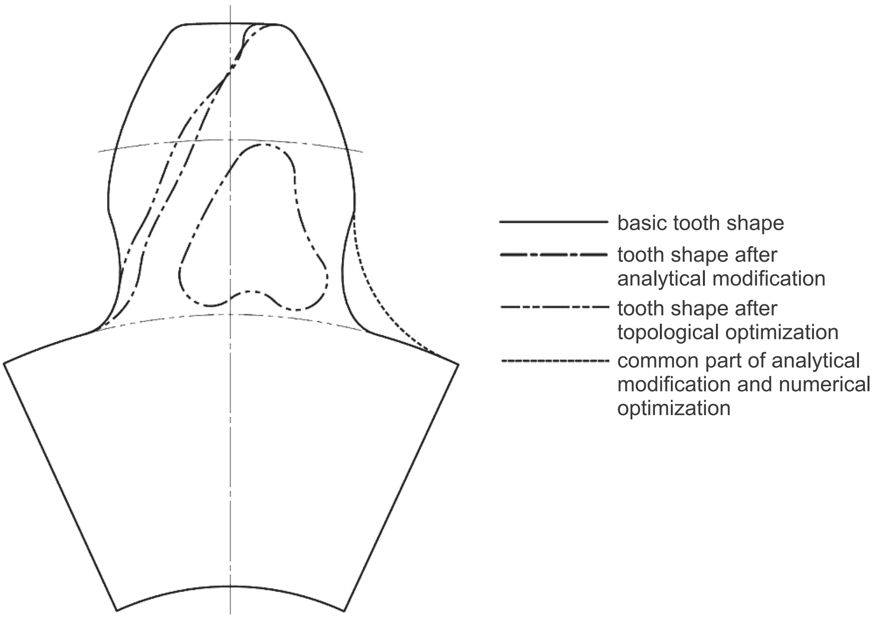

Fig. 6. Summary of the geometry of the obtained results

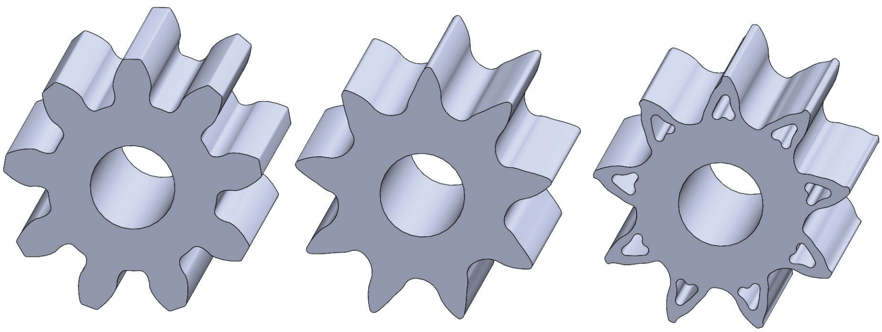

Fig. 7. Visualization of the obtained pinions

Conclusions

On the basis of the obtained results it may be concluded that the proposed methods of tooth modification allow to decrease the tooth mass and thus of the whole transmission. The presented methodology can be used wherever torque transmission occurs in one direction only, both in steel and polymer transmissions. Percentage weight reduction of 10-19% shown in the paper is not a big amount, but with polymeric gears used in appliances of the household sector, which are often produced in tens of millions, it may contribute to reduction of material purchase costs.

The methodology presented in the article allows for a more accurate estimate of the pressure ε_a (tooth contact ratio) for the analysed gears. Of course, only one gear was analysed, which does not allow for drawing general conclusions.

An interesting aspect is also the possibility of using a topological examination to reduce the mass of the tooth, described in chapter "Tooth modification based on topology optimization". Which, however, entails certain difficulties of the technological implementation of the obtained shape. Especially when making gears from steel materials. This would result in longer production times and would be associated with higher costs.

The analytical methodology for determining the shape of the tooth flank, described in Chapter "Tooth modification based on analytical calculations", which is not directly involved in the transmission of torque, allows for a relatively quick determination of the shape of the tooth flank. On the other hand, the disadvantage of the described methodology is the inability to derive universal dependencies for generating the shape of the tooth. In order to obtain the shape of the curves proposed in Figure 4, one should always use advanced computer systems (CAD), additionally supported by numerical calculations using, for example, Matlab software.

Another very important aspect of using the described methodology is creating solutions with lower weight while maintaining their original functionality. This can be used mainly in those designs where weight reduction is very important, such as: aircraft designs, racing cars and drone designs.

The proposed solution will also allow, in relation to steel gears, to reduce costs of finishing such as grinding or heat treatment. But in this matter the technology used in the above mentioned processes is very important, which sometimes determines the effect on both sides of the inter-tooth notch.

It should be emphasized that the proposed solution has not been experimentally verified, the proposed optimization should be read as a generally applied approach, and not as an appropriate solution for polymer gears. Nevertheless, the presented methodology is an introduction to further experimental studies, new geometries of gears made of polymer materials. The results of the above-mentioned research will be the subject of subsequent publications.

References

BERCZYŃSKI S., GRZĄDZIEL Z., RUKOWICZ S. 2016. *Analiza porównawcza naprężeń kontaktowych w zazębieniu przekładni zębatej napędu wału rozrządu silnika Sulzer RTA48T-B*. Zeszyty Naukowe Akademii Morskiej w Szczecinie, 10(82).

BRAUER J. 2002. *Analytical geometry of straight conical involute gears*. Mechanism and Machine Theory, 37.

BRECHER C., ROTHLINGSHOFER T., GORGELS C. 2009. *Manufacturing simulation of beveloid gears for the use in a general tooth contact analysis software*. Production Engineering, 3(1): 103-109.

CHERNETS M. 2019. *Method of calculation of tribotechnical characteristics of the metal-polymer gear, reinforced with glass fiber, taking into account the correction of tooth*. Eksploatacja i Niezawodność, 21(4).

DEPTULA A., PARTYKA M.A. 2018. *Application of complex game-tree structures for the Hsu graph in the analysis of automatic transmission gearboxes*. Journal of Machine Engineering, 18(4).

Dupont. 2021. Delrin® 500P NC010 ACETAL RESIN POM. https://dupont.materialdatacenter.com/en/products/datasheet/SI/Delrin%C2%AE%20500P%20NC010 (access: 20.09.2021).

DZIAMA A., MICHNIEWICZ M., NIEDŹWIEDZKI A. 1995. *Przekładnie zębate*. Ed. 2. PWN, Warszawa.

DZIUBEK T. 2018. *Application of coordination measuring methods for assessing the performance properties of polymer gears*. Polimery, 63(1).

HOMIK W., POŁOWNIAK P. 2012. *Podstawy konstrukcji maszyn – wybrane zagadnienia*. Wydawnictwo Politechniki Rzeszowskiej, Rzeszów.

KIŃSKI W., SOBIESKI W. 2020. *Geometry extraction from gcode files destined for 3d printers*. Technical Sciences, 23(2).

KISSsoft. 2021. https://www.kisssoft.com/en (access: 20.09.2021).

KLINGELNBERG. 2021. https://www.klingelnberg.de (access: 20.09.2021).

KOLLEK W., OSIŃSKI P., RUTAŃSKI J. 2015. *Wpływ zerowego luzu bocznego w zazębieniu na właściwości akustyczne pomp zębatych*. Napędy i Sterowanie, 1.

KROL O., SOKOLOV V. 2021. *Selection of worm gearing optimal structure for machine rotary table*. Diagnostyka, 22(1).

KURMAZ L.W., KURMAZ O.L. 2016. *Projektowanie węzłów i części maszyn*. Wydawnictwo Politechniki Świętokrzyskiej, Kielce.

LARSSON R. 2016. *Methodology for Topology and Shape Optimization: Application to a Rear Lower Control Arm*. Chalmers University of Technology, Goeteborg.

MAZANEK E. 2012. *Przyklady obliczen z podstaw konstrukcji maszyn*. Część druga. Wydawnictwa Komunikacji i Łączności, Warszawa.

MUMINOVIC A.J., COLIC M., MESIC E., SARIC I. 2020. *Innovative design of spur gear tooth with infill structure*. Bulletin of the Polish Academy of Sciences. Technical Sciences, 68(3).

Narzędzia profesjonalne EU. 2021. https://www.narzedziaprofesjonalne.eu/pl/p/Frez-monolityczny-LSM0202T-2%2C0-PAFANA-/2368 (access: 20.09.2021).

OCHĘDUSZKO K. 1985. *Koła zębate*. Ed. 8. Wydawnictwo Naukowo-Techniczne, Warszawa.

OSIŃSKI P. 2017. *Pompy zębate o obniżonym poziomie emisji hałasu*. Oficyna Wydawnicza Politechniki Wrocławskiej, Wrocław.

OSIŃSKI P., BURY P., CIEŚLICKI R., LOREK L. 2018. *Badania trwałościowe niskopulsacyjnej pompy zębatej*. Napędy i Sterowanie, 11.

PETR K., DYNYBYL V. 2014. *Optimization of gear-mesh single-stage gearbox for regional trains by FEM*. Zeszyty Naukowe Politechniki Śląskiej. Transport, 84.

POŁOWNIAK P., SOBOLAK M., MARCINIEC A. 2020. *Modelowanie wyjścia zwoju ślimaka globoidalnego z użyciem modyfikacji linii zęba*. Przegląd Mechaniczny, 12.

RATAJCZYK E. 2005. *Współrzędnościowa technika pomiarowa*, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa.

Shah C., Thigale S., Shah R. 2019. *Optimizing weight of a Gear using Topology Optimization*. International Journal of Science. Engineering and Technology Research (IJSETR), 7(6).

Sun K., Wang G., Lu Y. 2019. *Optimization method of bevel gear reliability based on genetic algorithm and discrete element*. Eksploatacja i Niezawodność, 21(2).

Tavčar J., Černe B., Duhovnik J., Zorko D. 2021. *A multicriteria function for polymer gear design optimization*. Journal of Computational Design and Engineering, 8(2): 581–599.

Twardoch K. 2014. *Cyfrowe modelowanie geometryczne zarysu zębów z zastosowaniem metodologii CAD*. Zeszyty Naukowe Politechniki Śląskiej. Transport, 82.

Tymczyszyn J. 2020. *The analysis of the gear's geometry measurement with various measuring systems*. Technologia i Automatyzacja Montażu, 3.

Wróbel I., Rysiński J. 2012. *Analiza strukturalna koła zębatego o rzeczywistym kształcie*. Mechanik, 85(2), CD.

Zelmer. 2021. ZMM4045B. https://zelmer.pl/produkt/zmm4045b/ (access: 20.09.2021).