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THE USE OF THE THEORY OF NONHOLONOMIC CONSTRAINTS IN THE PROCESS OF AUTOMATIC CONTROL OF A MANIPULATING MACHINE

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Abstract

The presented study contains a sample of utilization of the control laws treated as kinematic relations of parameter deviations and realized in the process of ordered automatic control of a manipulating machine. Movement of the grasping end is considered in an inertial reference standard rigidly joined with an immobile working environment of the manipulator. The specificity of the control's choice required creating program relations constituting the ordered parameters describing the movement of the manipulator's elements. During work, the ordered parameters are compared to the parameters realized in the process of the grasping end's work. This was deviations are determined, which thanks to properly prepared control laws are leveled by the manipulator's control executive system.

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Introduction

Manipulating machines due to their accuracy and repetitive positioning properties are nowadays an essential part of different industries such as industrial processes, medical fields, and automotive industries. They are used to assist in dangerous, monotonous, and tedious work. Typical example applications of manipulating machines in industry include moving, arranging, packaging, cutting, welding, paint spraying and sanding. Task accomplishment is a consequence of movements of manipulator's joints in single or multiple directions following a certain systematic pattern. The control variables are mainly position, velocity, force/torque or a hybrid combination of all.

The literature on dynamics modeling, control, application and performance analysis of manipulating machines is extensive. Systematic overview can be found in (AJWAD et al. 2015, JANKOWSKI 2005, NIZIOŁ 2005, SINGH, KRISHNA 2015) and many others. However, as shown in the article (AJWAD et al. 2015), it is a myth that the area of manipulator control is already saturated. The continuous advancement of science and technology and the improvement of people's living standards promote the continuous development and improvement of autonomous robot and manipulators technology. In the design and research of the robotic arm, the design of the control system is often inseparably related to the overall dynamic performance, especially for the control of the robotic arm joint (CAI et al. 2021, IVANOV et al. 2020, JANKOWSKI 2005, WEN et al. 2015). Robust path-planning and control algorithms are sought that must ensure the stability of all intermediate configurations of the manipulator along the prescribed path (BI 2020, SINGH, KRISHNA 2015). The method of incorporating constraint equations, holonomic and nonholonomic, into the kinematics or dynamics of the system at the modeling level was presented in papers (BERTONCELLI et al. 2020, JARZEBOWSKA, SANJUAN SZKLARZ 2017, KŁAK, JARZĘBOWSKA 2021, NEJMARK, FUFAJEW 1971). However, the available literature does not mention the introduction of control laws as nonholonomic constraints and their coupling with the dynamic equations of the manipulator motion. This research gap is filled by this study.

The studied issue is an innovative approach stemming from the previous works of the author (ŁADYŻYŃSKA-KOZDRAŚ 2009, 2012, SIBILSKA-MROZIEWICZ, ŁADYŻYŃSKA-KOZDRAŚ 2018) regarding modeling of dynamics and automatic control of flying objects. The positive results achieved within these works allow assuming that an analogical model of a controlled mobile object may be adopted with automatic control of the manipulating machine. The developed algorithm of the object control makes use of the complete nonlinear model of the object's dynamics combining it with control rights treated as nonholonomic constraints.

Thus, the purpose of this topic is to compile control rights treated as non-holonomic relations imposed on the manipulator's arms' movement. Mobile objects, such as manipulators and mobile robots, have a limitation of degrees of freedom through control. The imposed relations limiting free movement, which are non-holonomic relations, are considered control rights. It was assumed that a manipulator moves with program movement, thus the control rights were determined as geometric and kinematic relations of deviations between the ordered and actual trajectory of movement of the grasping end. This distinguish the presented algorithm from among other control algorithms, which can be found in polish and foreign literature.

Kinematic model – realized parameters of the manipulator's movement

When modeling of the movement of the manipulator frames of reference locating the object in space were assumed (Fig. 1). The primary frame of reference, in relation to which the manipulator's movements were considered, is the rigid inertial system related with its immobile foundation $O_1x_1y_1z_1$. The grasping end is related with the gravitational system $O_Cx_gy_gz_g$ with axes parallel to appropriate axes of the immobile system $O_1x_1y_1z_1$ as well as the grasping end's own system O_Cxyz. It is a dextrorotary system, with start in the C joint and axis O_Cx directed along the line connecting point C with the center of mass K of the grasping end along with the weight carried by it.

The study takes into account the example of a fictional manipulator, the model of which was selected in such a way, that it is possible to analyze a great variety of its kinematic pairs. And so (Fig. 1) two rotational parts were discriminated

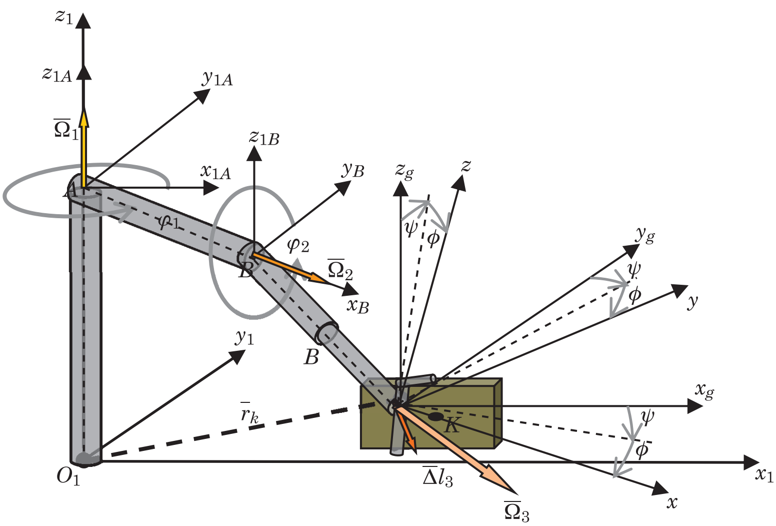

Fig. 1. Adopted frames of reference and locations as well as angular velocities of the elements manipulator

O_1A and AB $|\overline{O_1A}| = r_1 = \text{const.}, |\overline{AB}| = r_2 = \text{const.}$ rotations of which are described appropriately by angles φ_1 and φ_2 as well as angular velocities Ω_1 and Ω_2. The third arm – BC – is the moved arm ($|\overline{O_1A}| = r_1 = \text{const.}, |\overline{AB}| = r_2 = \text{const.}$), which can extend itself with angular velocity of Δl_3. However, the manipulator's grasping end, the length of which r_C is treated as the distance from point C to the center of mass K of the grasping end along with the weight carried by it, may perform any spherical movements in relation to joint C with angular velocity Ω_3 (Fig. 1). Spherical movements of the grasping end were described with the use of quasi-Euler movements φ, θ, ψ (NIZIOŁ 2005), combining the gravitational system $O_Cx_gy_gz_g$ with the grasping end's own system O_Cxyz.

These angles create kinematic relations imposed on the end

$$\bar{\Omega}_3 = \omega_x \bar{\imath}_1 + \omega_y \bar{\jmath}_1 + \omega_z \bar{k}_1$$

$$\omega_x = \dot{\varphi} - \dot{\psi}\sin\theta$$

$$\omega_y = \dot{\theta}\cos\varphi + \dot{\psi}\sin\varphi\cos\theta$$

$$\omega_z = -\dot{\theta}\sin\varphi + \dot{\psi}\cos\psi\cos\theta \tag{1}$$

Components ω_x, ω_y, ω_z of the temporary angular velocity of grasping end Ω_3 are linear relations of generalized velocities $\frac{d\varphi}{dt}, \frac{d\theta}{dt}, \frac{d\psi}{dt}$ with coordinates depending on the generalized coordinates φ, θ, ψ .

The kinematic relations imposed on the grasping end of the manipulator are described by dependencies formulated based on Figure 1.

The vector of temporary location of the working end in the system $O_1x_1y_1z_1$:

$$\bar{r}_K = \bar{r}_1 + \bar{r}_2 + \bar{l}_3 + \bar{r}_C = x\bar{\imath}_1 + y\bar{\jmath}_1 + z\bar{k}_1 \tag{2}$$

$$x = r_2\cos\phi_1 + (l_3 + \Delta l_3)\cos\phi_2\sin\phi_1 + x_C$$

$$y = r_2\sin\phi_1 - (l_3 + \Delta l_3)\cos\phi_2\cos\phi_1 + y_C$$

$$z = r_2 + (l_3 + \Delta l_3)\sin\phi_2 + z_C \tag{3}$$

The vector of temporary angular velocity of the end

$$\bar{\Omega}_K = \bar{\Omega}_1 + \bar{\Omega}_2 + \bar{\Omega}_3 = \Omega_x\bar{\imath}_1 + \Omega_y\bar{\jmath}_1 + \Omega_z\bar{k}_1 \tag{4}$$

$$\Omega_x = \Omega_2\cos\phi_1 + \omega_x$$

$$\Omega_y = \Omega_2\sin\phi_1 + \omega_y$$

$$\Omega_z = \Omega_1 + \omega_z \tag{5}$$

The vector of temporary linear velocity of the end

$$\bar{V}_K = \frac{d\bar{r}_K}{dt} = \bar{\Omega}_1 \cdot \bar{r}_1 + \bar{\Omega}_2 \cdot \bar{r}_2 + \frac{\delta \bar{l}_3}{\delta t} + \bar{\Omega}_3 \cdot \bar{r}_C = V_x \bar{\imath}_1 + V_y \bar{\jmath}_1 + V_z \bar{k}_1 \tag{6}$$

This way the kinematic relations, which provide information on the linear and angular location of the working end, realized during its work, were compiled. They were formulated in an inertial frame of reference $O_1x_1y_1z_1$ rigidly related with an immobile foundation of the manipulator and they constitute parameters realized by the automatically controlled manipulator.

Program relations – ordered parameters of the manipulator's movement

A necessary element of each control system is the targeting algorithm realized by it. Such an algorithm imposes boundaries on the object's movement. Thus, selection of the control method is very important.

In the case of a manipulator, in the situation, in which the grasping end is to perform a task specified in advance we assume, that the trajectory it should move along should be pre-ordered by the operator (Fig. 2). Thus it is a program movement, which is carried out along a spatial trajectory ordered in advance.

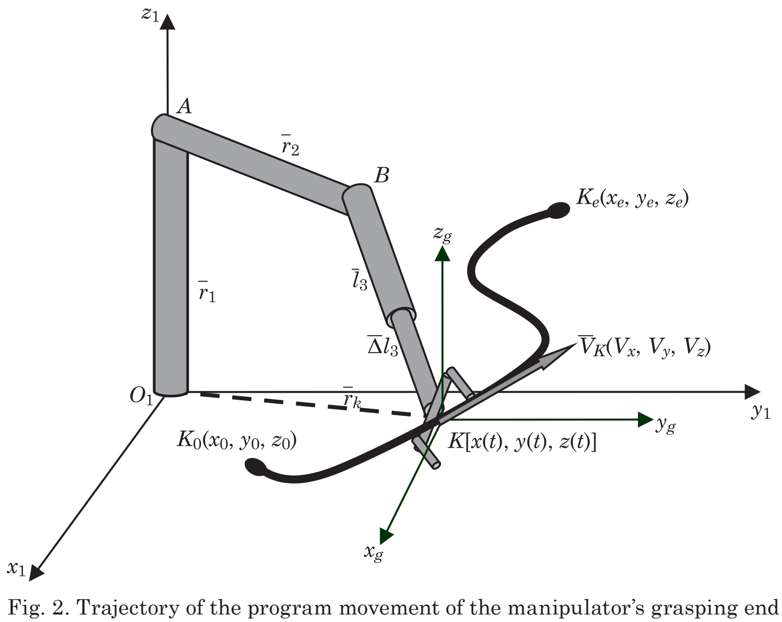

Fig. 2. Trajectory of the program movement of the manipulator's grasping end

Ordered program relations

$$\overset{\cap}{K_0 K_k} = s(x_z, y_z, z_z, t) \tag{7}$$

$$V_K(t) = \dot{s}(x_z, y_z, z_z, t) \tag{8}$$

where:

x_z,y_z,z_z – ordered parameters of the grasping end's movement.

In this case we are dealing, on one hand, with program geometrical relations (Eq. 7), which impose limitations on the spatial location of the manipulator's grasping end, and on the other hand with kinematic ties (Eq. 8), which impose limitations on its velocity vector – tangent to the ordered trajectory of movement. Thus the program ties constitute the ordered parameters of the manipulator's movement, which are compared to the parameters realized during targeting.

Control rights of the working end

Automatic control systems perform numerous types of tasks, which include: improvement of the dynamic properties of the controlled object, stabilization of the selected state parameters or directive changes of the ordered movement, performing the selected maneuvers, automatic realization of the object' complete movement along with all of its phases.

The proposed automatic control of the manipulating machine, according to the general concept show in the flowchart (Fig. 3), functions based on the previously calculated movement program and automatic stabilization utilizing the compiled control rights.

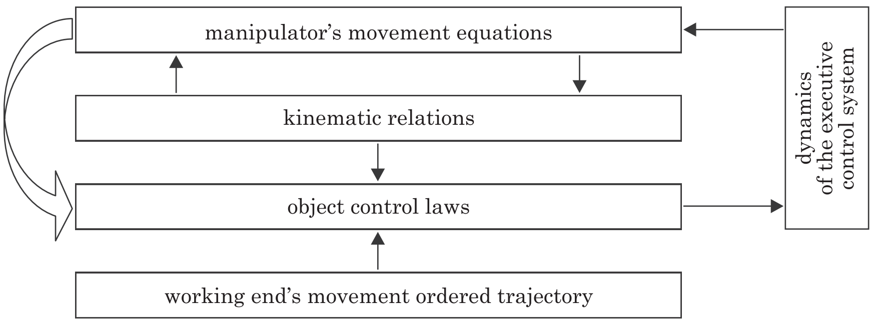

Fig. 3. Flowchart of the manipulator's control executive system

The control law were established in such a way that during movement of the manipulator deviations, which are the differences between the current state parameters (x, y, z, V_x, V_y, V_z, Ω_x, Ω_y, Ω_z) and the ordered parameters (x_z, y_z, z_z, V_x, V_y, V_z, Ω_x, Ω_y, Ω_z) are read. These deviations after proper amplification (with the use of K_j^i amplification coefficient) are transferred to the executive system of the control system, causing the ordered movements of the manipulator's arms.

$$T_1^{\phi_1}\dot{\phi}_1 + \phi_1 = K_{\Omega_z}^{\phi_1}(\Omega_z - \Omega_{zz}) + K_x^{\phi_1}(x - x_z) + K_y^{\phi_1}(y - y_z) + \phi_{10} \tag{9}$$

$$T_1^{\phi_2}\dot{\phi}_2 + \phi_2 = K_{\Omega_x}^{\phi_2}(\Omega_x - \Omega_{xz}) + K_{\Omega_y}^{\phi_2}\left(\Omega_y - \Omega_{yz}\right) + K_z^{\phi_2}(z - z_z) + \phi_{20} \tag{10}$$

$$T_1^{l_3}\dot{l}_3 + l_3 = K_{V_x}^{l_3}(V_x - V_{xz}) + K_{V_y}^{l_3}\left(V_y - V_{yz}\right) + K_{V_z}^{l_3}(V_z - V_{zz}) + l_{30} \tag{11}$$

where:
T_1^i – time constants,
K_j^i – control signals' amplification coefficients.

The set control laws are strongly non-linear functions of time. These are kinematic ties, non-integrable, not directly brought down to geometric ties, this is why they constitute non-holonomic ties imposed on the movement of the manipulator's arms (NEJMARK, FUFAJEW 1971). In order to verify such control laws the non-linear model of the manipulator's dynamics shall be adopted compiled with the use of analytical equations of movement for non-holonomic systems in generalized coordinates.

Conclusions

The study presents the kinematics model and control laws describing non-holonomic constraints imposed on the movement of manipulator's joints. The resulting equations are kinematic ties of deviations between the realized and ordered parameters of movement of the grasping end of the manipulator. They determine the relations between the kinematic parameters of the grasping end, and the possible deviations of the individual manipulator's joints, which are time-determined thanks to T^i time constants. This way the control laws, along with movement equations, determine the behavior of the manipulator on the route during tracking, in order to enable its optimum control. They allow to introduce automatic control of the grasping end moving along a programmed route.

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