



NUMERICAL SIMULATION OF FLOW THROUGH MICROCHANNELS WITH RANDOM ROUGHNESS

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Abstract

The aim of the study is to determine the effect of a randomly generated rough surface on the laminar flow of a fluid in a microchannel. Two-dimensional axially symmetric microchannels with a circular cross-section in the range of Reynolds number $Re = 100-1700$ were considered. Flow numerical simulations were performed using the Ansys / Fluent software.

Introduction

The rapid development of miniaturization observed in the last decade has provided new scientific problems and technological challenges. Microdevices and microcomponents are becoming increasingly widely used in numerous industrial applications from biomedicine to fuel cells. Most of these devices have microfluidic

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systems with various functionalities, from process to cooling (KMIOTEK, KUCABA-PIĘTAL 2018, KORDOS, KUCABA-PIĘTAL 2018, MARZEC 2021, ZAREMBA et al. 2018). Unfortunately, the direct translation of the flow models and heat transfer problems developed and tested for macrochannels is not justified for microchannels, because they do not take into account the phenomena significant in the microscale and the discrepancies increase with the reduction of the characteristic dimension and the surface treatment method. For example, as the channel size decreases, there is an increase in the effect of roughness on fluid flow.

While the flows in rough macrochannels are very well known, the micro scale has not been fully researched yet. Therefore, in recent decades, a lot of experimental research has been carried out on microchannels. Some of these studies indicate a change in the nature of the flow with a lower critical Reynolds number value than in the macro scale, as well as a greater roughness coefficient occurring in channels with a small hydraulic diameter. It is suggested by the increase of the roughness effect together with the decrease of the channel size (DAI et al. 2014, ZHANG et al. 2010).

The article presents how the flow in the microchannel is influenced by the occurrence of a randomly generated surface roughness, i.e., a set of unevenness (peaks and pits) on the real surface with relatively small spacing between vertices (KANDLIKAR 2006, WHITEHOUSE 2004),

In the macro scale, the material from which the element was made, the type of processing, and processing parameters undoubtedly have the greatest influence on the surface roughness. The surface can most often be characterized as a combination of two profiles – waviness and roughness (some surfaces also show a shape error). Waviness is measured over a much greater distance than roughness. The difference between the profiles is shown in Figure 1.

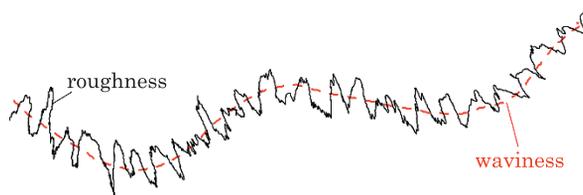


Fig. 1. Comparison of roughness and waviness

To describe the surface structure in the macroscale, various surface parameters are defined by means of numerical values. Many of them do not have a significant impact on the macroscale flow, however, they are particularly important in microchannels, because the considered inequalities are then larger. They can be divided into three groups (Fig. 2) describing the following features: amplitude, spacing, and slope, of which amplitude is of key importance in microflows.

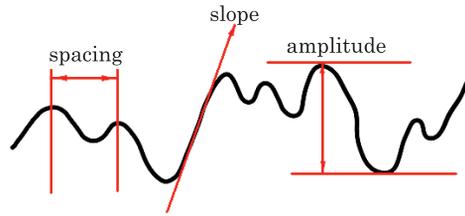


Fig. 2. Example parameters describing the surface

The most common and commonly used parameter describing the amplitude is the R_a parameter. Its value describes the arithmetic mean of the profile deviation from the mean. The calculation of this parameter is shown in Figure 3. The mean profile line (i) is determined by calculating the mean profile height along the sampling length. All negative deviations are converted to positive (ii). The mean line from the modified profile (iii) is then calculated. The R_a parameter alone is not sufficient to fully describe the surface roughness, but it is most commonly used in the case of microns. The R_a parameter is determined by the formula (WHITEHOUSE 2004):

$$R_a = \frac{1}{l_r} \int_0^{l_r} |z(x)| dx \tag{1}$$

where:

- l_r – sampling length [m],
- $z(x)$ – profile height along the length x [m].

Surface roughness is the result of the simultaneous interaction of many independent factors, both random and determined, and as a result it has a very complex microgeometry. The size of the surface roughness depends on the type of material and, above all, on the type of its processing. Roughness is caused,

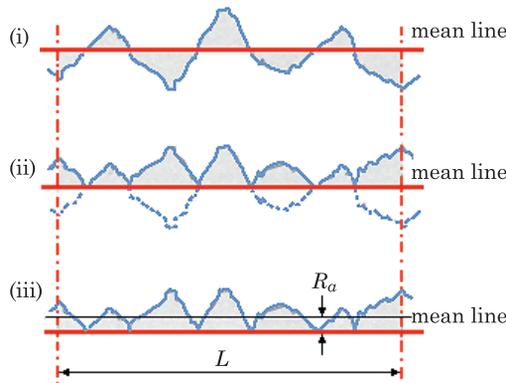


Fig. 3. R_a parameter

among others, by the processes of decohesion, plastic deformation in the cutting zone and formation of chip segments, friction of the tool contact surface against the machined surface, and chip friction against the machined surface (WHITEHOUSE 2004).

When machining surfaces, even on a microscale, surface roughness is inevitable. There are many types of micro-machining methods with machining precision from 10^{-2} μm to 5 μm , such as EDM (electrical discharge machining), ECM (electrochemical machining), pickling, micro-milling and so on (CHU et al. 2014). Depending on the production process, the microchannels can therefore even have a surface roughness in the order of the size of the channel. To properly design flow microdevices, it is necessary to determine the physical laws governing the fluid flow and heat transfer in the microchannel, taking into account the method of making the microdevice and the resulting roughness of the microchannel surface (KANDLIKAR 2006).

According to the knowledge of macrosystems, when the relative roughness is less than 5%, its influence on the coefficient of friction is negligible (KANDLIKAR 2006). For microscale channels, experimental and numerical results showed that surface roughness has a significant effect on heat transfer and heat transfer (ANSARI, ZHOU 2020, LU et al. 2020, MIRMANTO, KARAYIANNIS 2012). For example, the experiment of KANDLIKAR et al. (2005) indicated that for a 0.62 mm pipe with a relative roughness height of 0.355%, the influence of roughness on the coefficient of friction and heat conduction was significant.

In publication (DAI et al. 2014), the authors reviewed 33 scientific articles (a total of 5,569 data were collected) for flows in micro- and mini-channels with different wall roughness. The aim was to determine the effect of roughness on the friction coefficients. The authors concluded that if the relative roughness height is $<1\%$, it has little effect on the friction coefficient and the critical Reynolds number. A value of 1% has been suggested as the threshold for distinguishing between smooth and rough micro- and mini-channels.

Despite the availability of a large amount of experimental data, the flows in microscopes are still not fully investigated and described. Due to the scale of the phenomenon, numerical simulations seem to be a good tool. The data obtained through computational methods allows the analysis of many aspects difficult to grasp in an experiment, which will enable understanding of the basic physics of the problem.

Most often in the literature, the surface roughness is modeled with simple geometric shapes, e.g., triangles, rectangles, squares, ellipses, trapeziums (LALEGANI et al. 2018, ZHANG et al. 2010, CROCE 2016) and the fractal geometry method (JIA, SONG 2019) or Gaussian function (PELEVIĆ, VAN DER MEER 2016). However, due to the stochastic nature of roughness, a better approach seems to be to randomly generate surface irregularities, which will be described later in the article.

Literature analyzes show that roughness affects the streamline distribution. With a high roughness value, this can lead to the flow being detached near the wall and the formation of recirculation zones. Detachment of flow and recirculation areas are most likely the main causes of increased friction and the formation of a pressure gradient (ZHANG et al. 2010).

The aim of the study was to estimate the effect of roughness on the laminar flow of fluid in microchannels, as well as to select an appropriate method of roughness modeling to achieve the best compliance with the experimental data.

Research methodology

Two-dimensional axially symmetrical microchannels with a circular cross-section were considered. On the wall of the microchannel there are elements simulating the roughness of the channel generated on the basis of the normal distribution, with the density function:

$$f_{\mu,\sigma}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x - \mu)^2}{2\sigma^2}\right) \tag{2}$$

where:

- μ – expected value [–],
- σ – standard deviation [–],
- x – a random variable with the expected value μ and variation σ^2 [–].

The area was obtained by generating a random value from 0 to 1 as a random variable. The average was the radius of the channel (i.e., 25 μm), while the standard deviation was defined as the product of 1.25 and R_a (ZHANG et al. 2010). The radius r_i is randomly generated by the above-mentioned function. The variable x_i changed in the axial direction by a constant value equal to S_a ($x_{i+1} = x_i + S_a$). Having these 2 parameters, it was possible to generate a random (one-dimensional) curve. To achieve the three-dimensional effect, further curves with S_ϕ spacing are generated. The microchannel is modeled as axisymmetric, therefore the curves are averaged (i.e., rays having the same x variable are averaged).

As a result, we get i points with radius r_i and location on the x_i axis. This is visualized in Figure 4. Due to the randomness of the process, R_a of each surface is monitored, then the mean R_a of the generated surfaces is calculated. The final surface is accepted if the mean R_a (calculated from all surface components) does not differ by more than 0.3% from the target R_a .

The flow was assumed to be two-dimensional, axisymmetric, incompressible, and steady. The diameter of the channel was a characteristic dimension for the Reynolds number. The test fluid was water at room temperature (fluid density $\rho = 998 \text{ kg/m}^3$, dynamic viscosity $\mu = 0.001 \text{ Pa}\cdot\text{s}$). The microchannel geometry is presented in Figure 5.

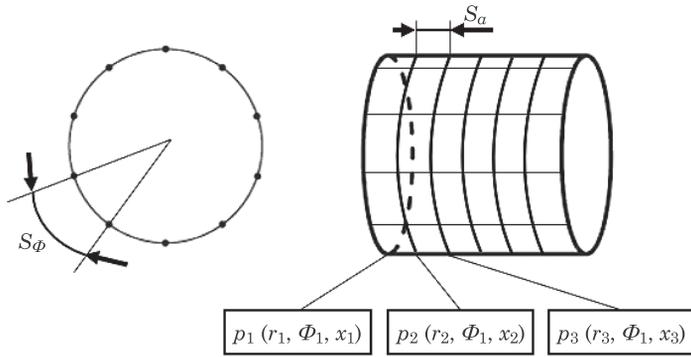


Fig. 4. The method of generating random points

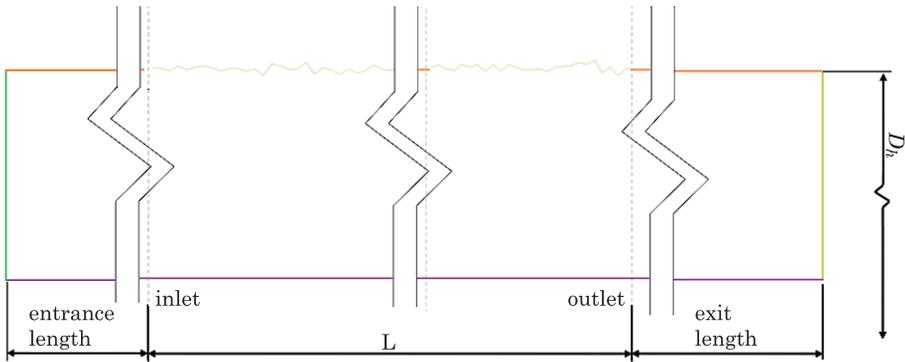


Fig. 5. Geometry of the microchannel

The parameters adopted for the calculations are presented in Table 1, while the parameter values used for the modeling of the random surface are presented in Table 2.

Parameters used in the calculations

Table 1

Characteristic	Symbol	Value	Unit
Microchannel length	L	1	mm
Microchannel diameter	D_h	50	μm
Inlet velocity	V_{in}	2,01-42,12	m/s

Parameters used in the modeling of a random surface

Table 2

S_ϕ	S_a
36°	1 μm

The flow in the microchannel results from the principles of fluid conservation (KANDLIKAR 2006):

1. Principles of conservation of mass:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \bar{V}) = 0 \tag{3}$$

2. Principles of conservation of momentum:

$$\rho \frac{\partial \bar{V}}{\partial t} = \rho \bar{F} - \text{grad } p + \mu \Delta \bar{V} \tag{4}$$

where:

- \bar{V} – velocity vector [m/s],
- ρ – fluid density [kg/m³],
- \bar{F} – vector of mass forces [N],
- p – fluid pressure [Pa].

The system of equations adopted for modeling can be written in the form of equations:

– equation of continuity:

$$\frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{\partial (u_z)}{\partial z} = 0 \tag{5}$$

– momentum equations (Navier-Stokes):

$$\begin{aligned} \rho \left(V_r \frac{\partial V_r}{\partial r} + V_z \frac{\partial V_r}{\partial z} \right) &= -\frac{\partial p}{\partial r} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V_r}{\partial r} \right) - \frac{V_r}{r^2} + \frac{\partial^2 V_r}{\partial z^2} \right] \\ \rho \left(V_r \frac{\partial V_z}{\partial r} + V_z \frac{\partial V_z}{\partial z} \right) &= -\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V_z}{\partial r} \right) + \frac{\partial^2 V_z}{\partial z^2} \right] \end{aligned} \tag{6}$$

The commercial ANSYS Fluent software, using the finite volume method, was used to solve the equations.

The boundary conditions adopted for the analysis are:

- at the inlet to the duct, a profile was set up at the outlet of the reference pressure,
- zero tangential velocity on the channel wall (also impenetrability of the walls),
- axial symmetry.

The Reynolds number was determined from the formula (KANDLIKAR 2006):

$$\text{Re} = \frac{\rho V D_L}{\mu} \tag{7}$$

where:

- D_L – characteristic dimension [m],
- μ – dynamic viscosity [kg/m·s],
- V – flow velocity [m·s].

To determine the effect of roughness on the flow in the microchannel, the formulas used in macrochannels and milichannels were used. The dimensionless coefficient of friction (Fanning, Dracy) was determined (KANDLIKAR 2005). The Fanning friction coefficient (f_F) is defined as:

$$f_F = \frac{\tau_w}{\frac{1}{2} \rho u_m^2} \quad (8)$$

where:

- τ_w – tangential stresses [Pa],
- u_m – average velocity of the liquid in the channel [m/s].

The Darcy's coefficient of friction (f_D), related to the Fanning's coefficient of friction, is expressed as:

$$f_D = 4f_F \quad (9)$$

Darcy's coefficient is also defined as the ratio of the Poiseuille number P_o to the number Re as in the formula (KANDLIKAR 2006):

$$f_D = \frac{P_o}{\text{Re}} \quad (10)$$

The Poiseuille number assumes a constant value, in the case of developed laminar flow, it differs depending on the shape of the channel cross-section. For a circular channel, the Poiseuille number is assumed to be constant at 64.

Formula for pressure drop taking into account friction losses:

$$\Delta p = \frac{2f_D \rho u_m^2 L}{D} \quad (11)$$

where:

- D – channel diameter or hydraulic diameter if the channel cross-section is other than circular [m],
- L – channel length [m].

The Dracy coefficient of friction depends on: the type of flow, wall roughness, pipe geometry (length, diameter) and is most often determined using the Moody diagram. Dracy's coefficient of friction for laminar flow can be determined based on the Hagen-Poiseuille law (KANDLIKAR 2005):

In macro and microscale in laminar flow, the coefficient of frictional loss depends on the Reynolds number and not on the roughness (KANDLIKAR 2005).

According to (DAI et al. 2014) after calculating the coefficient f_D , it is possible to calculate the corrected roughness coefficient matching the experimental data obtained in microchannel tests using, for example, the Gaussian distribution model, which assumes that the roughness behaves in accordance with the Gaussian normal distribution. The correlation was developed based on the standard deviation of the roughness profile. A correction factor has been introduced that modifies the friction coefficient f_D . The relative roughness of the discussed correlation is:

$$\frac{f_{Dcorr}}{f_D} = \begin{cases} 1 / \left[1 - 23 \left(\frac{2.5\varepsilon}{D} \right)^2 \right] & \text{for } \varepsilon/D \leq 0.04 \\ 1 / \left[1 - 50 \left(\frac{2.5\varepsilon}{D} \right)^{2.4} \right] & \text{for } 0.04 \leq \varepsilon/D \leq 0.06 \end{cases} \quad (12)$$

where:

ε – the roughness height of the inner surface of the microchannel [m].

The construction of the microchannel geometry takes into account the entrance length. The entrance length was calculated from the formula (KANDLIKAR 2006):

$$h/D \cong 0.05 \text{ Re} \quad (13)$$

The entrance length of 6 mm was adopted for $\text{Re} = 2,100$. The value of the exit length was set at 0.5 mm.

Grid Convergence Index (GCI) relative to the average velocity in the section located in the middle of the channel length was determined to assess the correctness of the mesh compaction:

$$\text{CGI} = F_s \cdot \frac{\left| \frac{u_{h2} - u_{h1}}{u_{h1}} \right|}{a^p - 1} \cdot 100 \quad (14)$$

where:

F_s – security factor [-],

u_{h1}, u_{h2} – selected parameter (velocity is assumed) [m/s],

h_1, h_2 – number of finite elements [-],

$p = \frac{h_2}{h_1}$ – współczynnik zagęszczenia siatki [-],

a – the approximation order of the calculations (the assumed value is 2) [-].

In order to optimize the mesh, a study of the GCI coefficient was performed. These studies have shown that with an element size of 0.1 μm , the GCI coefficient is less than 0.4%.

The mesh was generated from elements of the prism and tetrahedron type.

Research results

To investigate the effect of roughness on the flow in the microchannel, the following were analyzed:

- changing the shape of the roughness element – as a surface composed of points that meet the Gaussian distribution for the flow of fluid in a channel with a circular cross-section,
- a change in the Reynolds number from $Re = 100$ to $Re = 1,700$.

Based on the calculations, the coefficients of linear loss, pressure loss were determined, and the current lines and velocity vector distributions were generated. Linear loss coefficients were compared to those derived from classical theory, Gaussian random distribution, and experimental data (MOHIUDDIN MALA, LI 1999).

The effect of the scatter of results resulting from the pseudo-random distribution of points from which the geometry is built was also investigated.

Due to the use of randomly generated geometry, it was required to create a statistical sample. For this purpose, 30 sample random surfaces of $R_a=1.75 \mu\text{m}$ (Fig. 6). The Darcy coefficient (f_{rand}) was calculated from the formula (11), on the basis of the mean of these 30 models, for Reynolds numbers 100-1,700. Then, the standard deviation was calculated (Tab. 3).

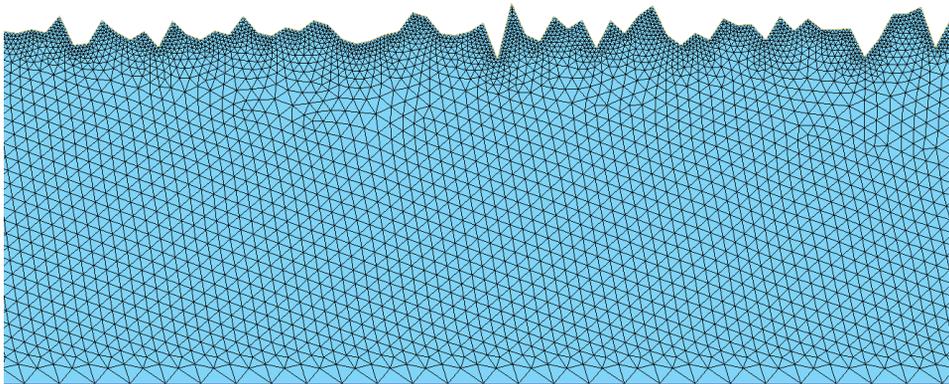


Fig. 6. An example of one of the generated geometries, together with the mesh

The results in Table 3 show that the greatest uncertainty concerns the higher values of Reynolds number, for $Re = 1,700$ -99.8% of the results are within $\pm 5.3\%$ of the mean value, while for $Re = 100$ only within $\pm 2.1\%$.

Moreover, a comparison the Darcy roughness coefficient was made of the results from the random surface simulation (f_{rand}), the coefficients resulting from the classical theory ($f_{\text{classic}} = 64/Re$) and the results of the experimentally

Table 3

The influence of randomness on the results of the analyzes

Re	100	300	500	900	1,300	1,700
V	2.01	6.03	10.05	18.09	26.13	34.17
f_{rand}	0.704	0.241	0.149	0.090	0.068	0.056
σ	0.0051	0.0023	0.0018	0.0014	0.0013	0.0011
$f_{rand} + 3\sigma$	0.719	0.248	0.155	0.094	0.072	0.059
$f_{rand} - 3\sigma$	0.689	0.234	0.144	0.086	0.064	0.053

obtained coefficients (f_{exp}) (MOHIUDDIN MALA, LI 1999), as well as those generated by the Gaussian random distribution formula ($f_{gaussian}$) calculated from the formula (12). In the equation, f_D has been replaced by $f_{classic}$. All results were compared to the experimental data (Tab. 4).

Table 4

Comparison of Darcy roughness coefficients

Re	100	300	500	900	1,300	1,700
V	2.01	6.03	10.05	18.09	26.13	34.17
f_{exp}	0.720	0.242	0.146	0.088	0.068	0.058
f_{rand}	0.704	0.241	0.149	0.090	0.068	0.056
Absolute difference	-0.016	-0.001	0.003	0.002	0.000	-0.002
Relative difference	2.2	0.4	1.9	2.1	0.3	4.1
$f_{classic}$	0.640	0.213	0.128	0.071	0.049	0.038
Absolute difference	0.080	0.028	0.018	0.017	0.018	0.021
Relative difference	11.1	11.7	12.6	19.5	27.3	35.5
$f_{gaussian}$	0.777	0.259	0.155	0.086	0.060	0.046
Absolute difference	-0.057	-0.017	-0.009	0.002	0.008	0.013
Relative difference	7.9	7.2	6.1	2.3	11.7	21.7

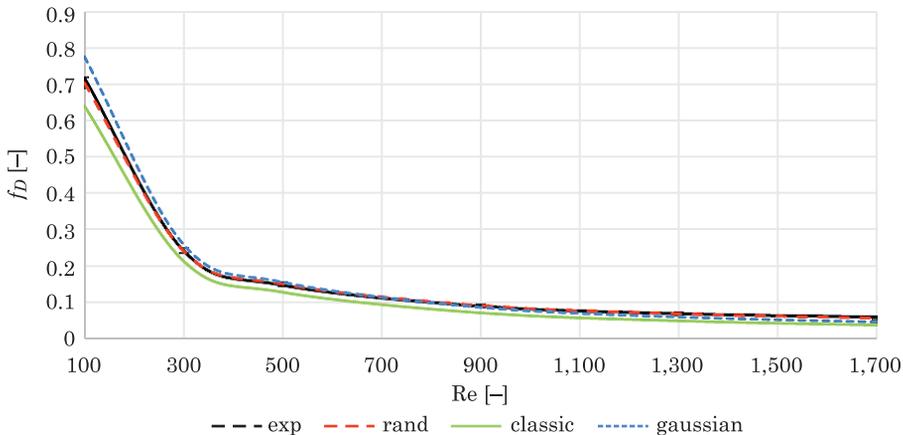


Fig. 7. Comparison of roughness coefficients

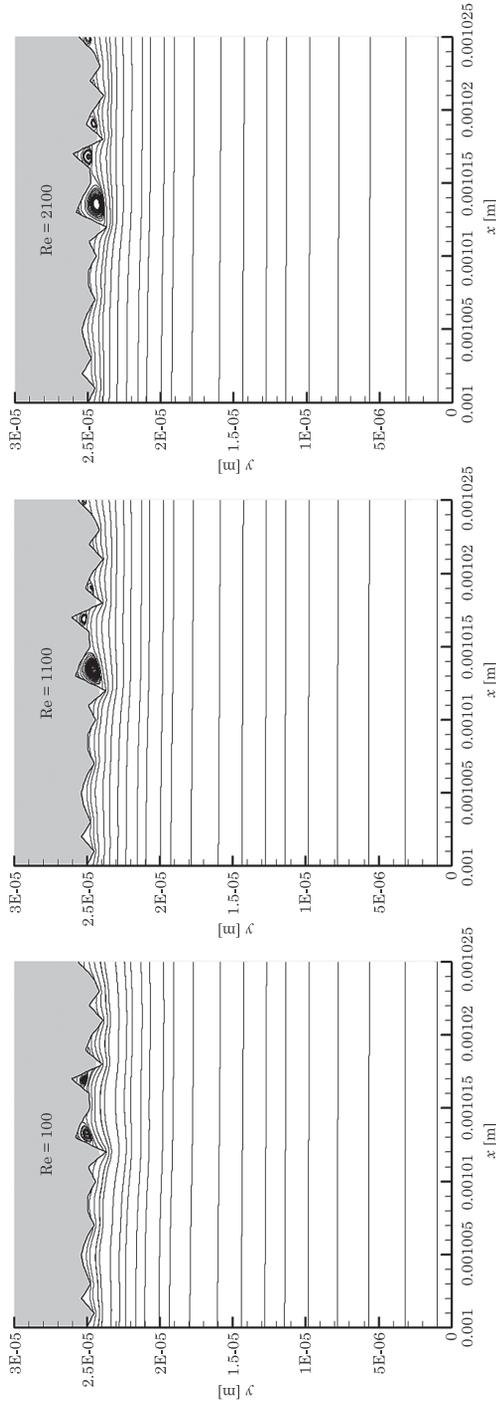


Fig. 8. Distribution of streamlines for a randomly generated surface

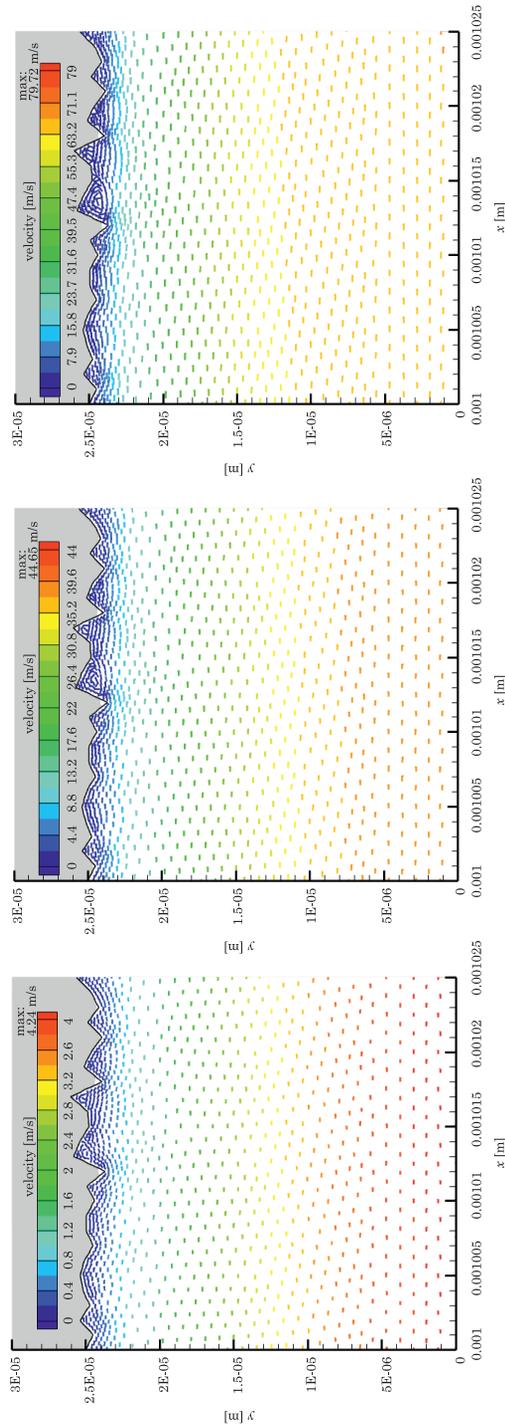


Fig. 9. Distribution of velocity vectors for a randomly generated geometry

Based on the results contained in Table 4 and Figure 5, it can be concluded that the randomly generated surface is very consistent with the experimental results. The maximum relative difference is about 4% for Reynolds number 1,700. For smaller Reynolds numbers the error does not exceed 2.2%. Using the data from the classical theory, it is possible to compute the corrected coefficients of a correlation using a random Gaussian distribution. It gives relatively good results not exceeding 8% error for Reynolds numbers from 0 to 1,100.

To visualize the flow and present the influence of the Reynolds number on the flow, the distributions of the streamlines and velocity vectors for the Reynolds numbers 100, 1,100 and 2,100 were generated for an exemplary configuration selected from 30 models (Fig. 6, 7).

The results of flows in microchannels with roughness w presented in Figures 8 and 9 show the deformation of the flow image. The analysis of the test results shows the formation of circulation zones in the cavities between the elements. With increasing speed, they increase their share in the considered cavity (the length of the vortex increases). The distribution of the vortices is irregular, depending on the depth of the pit and the Reynolds number.

To investigate the pressure drop in the microchannel, 26 measurement planes have been defined. The pressures were related to the cross section at the inlet to the channel. The inlet velocity is 2.01 m/s ($Re = 100$) and 22.11 m/s ($Re = 1,100$). R_a is $1.75 \mu\text{m}$. The results are presented for 3 exemplary models (surf_1, surf_2, surf_3) selected from the group of 30 created (Fig. 8, 9).

The pressure drop for $Re = 100$ is close to the linear function. However, when the flow velocity increases to $Re = 1,100$, pressure fluctuations along the channel axis become apparent. The nature of the curve is similar, but the larger scale (for $Re = 1,100$) allows to see the nonlinearity of the phenomenon. Despite the fact that the pressure drop has a completely different character for different models, they all converge to one value for $l = 0.001$ m.

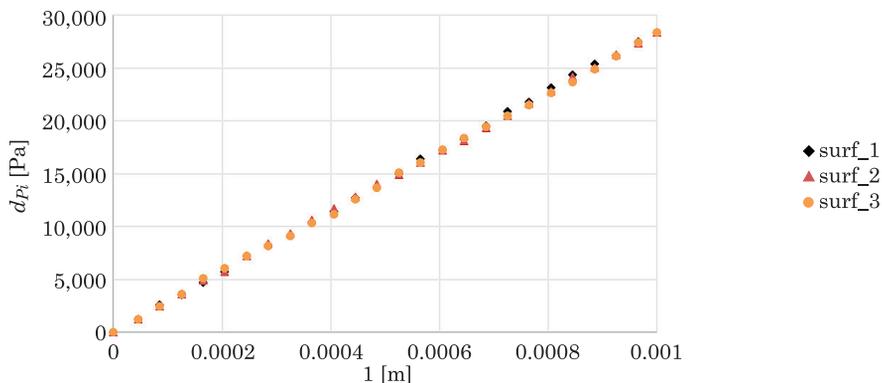


Fig. 10. Pressure drop for the number $Re = 100$

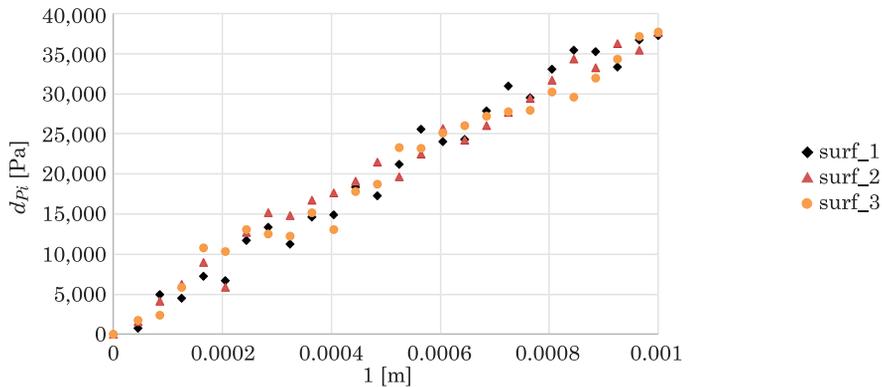


Fig. 11. Pressure drop for the number $Re = 1,100$

Conclusions

Along with the development of miniaturized technologies, microchannels have found wide application in microdevices, including in cooling systems of various technical devices, especially electronic devices. Hence, the need to research the influence of roughness on the flow in microchannels. The article presents the results of numerical tests of the influence of randomly modeled roughness on the flow and friction losses in microchannels. The analysis of the obtained results showed that the roughness has a significant influence on the flow in the microchannel.

The most important conclusions are as follows:

- Using the method of generating random geometry presented in the article, a very good agreement with the experimental data was obtained, especially in the range of lower Reynolds numbers, where the relative error was only 2.1%.
- The main reason of observed pressure drop is formation of recirculation zones in the roughness pits.
- The one of parameters influencing the length of the recirculation zones are the roughness height. As a result of increasing the speed, the vortices increase with the filling of a given cavity.
- Friction losses decreases as the Reynolds number increases.
- Comparing several methods of calculating the Darcy roughness coefficient, it was found that generating random geometry is the method whose results are closest to the data obtained by experiments. The coefficients resulting from the classical theory generate from 11.1 to 35.5% error (depending on the Reynolds number), therefore it is not the recommended method for calculating the linear loss factor in microchannels. On the other hand, the corrected coefficients using the formula obtained from the Gaussian distribution model achieve an amazingly

good result from 2.3 to 8% (for Reynolds numbers in the range 100-1,100). The discussed model works worst for Reynolds numbers in the range 1,100-1,700, reaching 22% of the relative error.

– The model created on the basis of pseudorandom numbers highlights the nonlinearity of the pressure drop along the channel axis (for higher Reynolds numbers – eg $Re = 1,100$). This is expected on a micro scale where there are abrupt changes in parameters due to an increased proportion of roughness relative to the geometry dimension.

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